Plumbing Theory
MANUSCRIPT

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Contents

Chapter 1

Plumbing Theory

Plumbing theory refers to any nonphysical aspect of plumbing. Any time spent in the classroom, looking at blueprints, estimating developed length, sketching pipes on a napkin, etc. - falls under plumbing theory.

Plumbers in working Massachusetts learn plumbing theory predominantly from the State *Plumbing Code* to ensure the safe and adequate operation of finished plumbing systems. Formally called *248 CMR 10.00*, the *Uniform State Plumbing Code* is available via *mass.gov* as PDF. Edition 12/8/23 is used for the majority of this study.

This study does not attempt to supplant or displace the myriad resources available to student and established plumbers. Nor does it attempt to convey any baseline understanding of any plumbing principle. For this, it is only appropriate to begin with Newton's laws and a study of thermodynamics and fluids.

Instead, the purpose here is to articulate certain parts of the standard plumbing literature that are lacking in adequate detail or lacking in explanation altogether.

1 Fixture Units

1.1 Review of Definitions

A term used heavily throughout the plumbing code is the *fixture unit*, which has a definition tailored for plumbers appearing in *Section 10.03: Definitions*:

Fixture Unit. One cubic foot of water drained in a 1.25 inch pipe over a period of one minute. One cubic foot of water is equal to 7.5 gallons.

The definition provided is alarmingly vague. Whether (or not) one cubic foot of water really drains through a 1.25 inch diameter pipe in precisely one minute is questionable. Needless to mention, one cubic foot is closer to 7.4813 gallons of liquid. As 7.5 has two digits of precision, the writer may as well have stated the pipe is 1.3 inches in diameter.

Dimensionality

Given the definition, one may expect that a fixture unit is a flow rate, perhaps measured in gallons per minute, and then perhaps multiplied (or divided?) by 1.25 somewhere, because surely the pipe diameter factors in. It turns out none of this is correct.

Browsing CMR 10.00 or comparable resources, one instead finds that fixture units are *not* measured in physical units, but are instead always dimensionless scale factors like 3, π, or 1000. This is also troublesome for the definition, because we lack three numbers to balance the units of gallons, minutes, and inches.

Also found in *Section 10.03: Definitions* is the definition of *load factor*, which reads:

Load Factor. The percentage of the total connected fixture unit flow which is likely to occur at any point in the drainage system. It varies with the type of occupancy, the total flow unit above this point being considered, and with the probability factor of simultaneous use.

This definition is a slight improvement over the previous one by alluding to likelihood of occurrence and type of occupancy. Sadly, the terms 'probability' and 'load factor' appear nowhere else in CMR 10.00.

Reaching for the *Uniform Plumbing Code*, *IAMPO/ANSI 1 - 2021*, one finds an honest yet vague word salad explaining the fixture unit:

Fixture Unit. A quantity in terms of which the load-producing effects on the plumbing system of different kinds of plumbing fixtures are expressed on some arbitrarily chosen scale.

We finally encounter a sensible definition of the fixture unit in the *International Plumbing Code*, *ICC A117.1-2017*, which reads:

Drainage Fixture Unit. A measure of the probable discharge into the drainage system by various types of plumbing fixtures. The drainage fixture-unit value for a particular fixture depends on its volume rate of drainage discharge, on the time duration of a single drainage operation and on the average time between successive operations.

One can speculate as to why the definition of the fixture unit varies so vividly per authority. Regardless of this, any literal understanding of the matter will remain elusive if plumbing codes are the only resource.

The Plan

We've racked up plenty of technical debt by skewering the definition of the fixture unit, and the intent here is not to solve the problems with the definition stated in CMR 10.00 or other publications. Instead, the plan is to take a first-principles approach using probability theory in the same way as done by the inventor of the fixture unit, Roy B. Hunter, in the years leading to 1940.

We start with a homemade warm-up exercise called the **Valve and Tank Problem**. This problem is informed by a mixture of real data and more-or-less made up numbers corresponding to likeliness of occurrence, type of occupancy, etc., of certain plumbing fixtures.

Next, we solve a problem called **Hunter's Calculation** to once-and-for-all force an understanding of the fixture unit. Such an effort also yields the so-called *Hunter's curve*, which is the source of the various 'gallons per minute versus fixture unit' tables occurring in the myriad of plumbing resources, including CMR 10.00.

Fixture unit values tend to change over time and location, however the work that follows will readily generalize to suit any fixture parameters. Data contemporary to the 1940s is used to ensure we're on track with the original work of R. B. Hunter.

1.2 Valve and Tank Problem

Two devices, or *fixtures* in American plumbing systems are the (i) *flush valve*, and (ii) the *flush tank*. The flush valve conveys 4 gallons of water over a period of 9 seconds.[1] The flush tank conveys 4 gallons in 60 seconds. Assume that all flush valves and flush tanks are used six times per hour on average.

If a certain building has $V = 20$ flush valves and $T = 30$ flush tanks, use only the information provided to (i) estimate the combined number of devices j being used simultaneously. (ii) Calculate the probability $P(j)$ of any j occurring and check that the estimate for j is correct. (iii) Calculate the probability that zero devices are in use at a given moment. (iv) Determine the number k at which $P(k) \approx 1\%$. (v) Calculate the final water demand using k.

Per hour (3600 seconds), a flush valve is operating for an average of 54 seconds. Similarly, a flush tank operates for 360 seconds. At a given moment, there is a

$$p_v = \frac{54}{3600} = 0.015$$

probability that a given flush valve is operating, and a

$$p_t = \frac{360}{3600} = 0.1$$

probability that a flush tank is operating.

[1] Figures gathered from PDH Course M126, *Sizing Plumbing Water System.* A. Bhatia. 2020. *www.PDHonline.prg*

Modal Analysis

Let x_v denote the number of flush valves in use simultaneously, and let x_t be the number of flush tanks being used such that

$$j = x_v + x_t$$

at a given moment.

Modeling each fixture as a weighted coin, we know from the binomial distribution that the most probable values for x_v, x_t are approximately the respective modes

$$\begin{aligned} x_v^* &= V p_v - (1 - p_v) \approx V p_v \\ x_t^* &= T p_t - (1 - p_t) \approx T p_t \ , \end{aligned}$$

or

$$\begin{aligned} x_v^* &= (20)\,(0.015) = 0.3 \\ x_t^* &= (30)\,(0.1) = 3 \ . \end{aligned}$$

The convoluted mode j^* ought to be the sum of the individual modes

$$j^* \approx x_v^* + x_t^* \ ,$$

and using the numbers on hand, one finds

$$j^* \approx 3.3 \ .$$

That is, we expect about three devices to be conveying water at a given moment. The total mode is essentially dominated by x_t^*.

In terms of x_v, x_t, the water demand rate $D\,(x_v, x_t)$ is given by

$$D\,(x_v, x_t) = x_v \left(\frac{4\text{ gal}}{9\text{ s}}\right) + x_t \left(\frac{4\text{ gal}}{60\text{ s}}\right) \ .$$

Using $j = 3$, it follows that x_v can take on any value 0, 1, 2, 3. Thus:

$$\begin{aligned} D\,(0,3) &= 0.2\text{ gal/s} \\ D\,(1,2) &= \left(0.44\overline{4} + 0.13\overline{3}\right)\text{ gal/s} = 0.57\overline{7}\text{ gal/s} \\ D\,(2,1) &= \left(0.88\overline{8} + 0.066\overline{6}\right)\text{ gal/s} = 0.95\overline{5}\text{ gal/s} \\ D\,(3,0) &= 1.3\overline{3}\text{ gal/s} \end{aligned}$$

To make use of the above information, recall from the values of x_v^*, x_t^* that $(0,3)$ (zero flush valves in use, three flush tanks in use) is the most likely configuration at a given moment, and the typical water demand is estimated at 0.2 gal/s, or 12 gallons per minute.

Of course, we don't want to design based on typical use. It's much better to anticipate the worst-probable case, which is not the worst *possible* case. (To design based on the worst possible case scenario is inefficient, costly, or worse.)

Probabilistic Analysis

The probability that there are x_v flush valves in use is given by the binomial distribution:

$$P_v\,(x_v) = \binom{V}{x_v} (1 - p_v)^{V - x_v}\, p_v^{x_v}$$

Also in terms of x_v, we write for the flush tank:

$$P_t\,(j - x_v) = \binom{T}{j - x_v} (1 - p_t)^{T - (j - x_v)}\, p_t^{j - x_v}$$

The total probability is the sum of convolutions of the two above distributions:

$$P\,(j) = \sum_{x_v = \max(0, j-T)}^{\min(j,V)} P_v\,(x_v) \cdot P_t\,(j - x_v)$$

Using $j = 3$, the above probability is

$$P\,(3) = \sum_{x_v=0}^{3} P_v\,(x_v) \cdot P_t\,(j - x_v) \ ,$$

where:

$$\begin{aligned} P_v\,(x_v) &= \binom{20}{x_v} (1 - 0.015)^{20 - x_v}\,(0.015)^{x_v} \\ P_t\,(j - x_v) &= \binom{30}{3 - x_v} (1 - 0.1)^{30 - (3 - x_v)}\,(0.1)^{3 - x_v} \end{aligned}$$

Evaluating $P\,(3)$ is quite a chore. For $x_v = 0$, we have:

$$\begin{aligned} P_v\,(0) &= \binom{20}{0} (1 - 0.015)^{20}\,(0.015)^0 \approx 0.7391 \\ P_t\,(3) &= \binom{30}{3} (1 - 0.1)^{27}\,(0.1)^3 \approx 0.2361 \\ P_v\,(0) \cdot P_t\,(3) &\approx 0.1745 \end{aligned}$$

Continuing for $x_v = 1$:

$$\begin{aligned} P_v\,(1) &= \binom{20}{1} (1 - 0.015)^{19}\,(0.015)^1 \approx 0.2251 \\ P_t\,(2) &= \binom{30}{2} (1 - 0.1)^{28}\,(0.1)^2 \approx 0.2277 \\ P_v\,(1) \cdot P_t\,(2) &\approx 0.05126 \end{aligned}$$

Continuing for $x_v = 2$:

$$\begin{aligned} P_v\,(2) &= \binom{20}{2} (1 - 0.015)^{18}\,(0.015)^2 \approx 0.03257 \\ P_t\,(1) &= \binom{30}{1} (1 - 0.1)^{29}\,(0.1)^1 \approx 0.1413 \\ P_v\,(2) \cdot P_t\,(1) &\approx 0.004602 \end{aligned}$$

Continuing for $x_v = 3$:

$$P_v(3) = \binom{20}{3}(1-0.015)^{17}(0.015)^3 \approx 0.002976$$
$$P_t(0) = \binom{30}{0}(1-0.1)^{30}(0.1)^0 \approx 0.04239$$
$$P_v(3) \cdot P_t(0) \approx 0.0001262$$

The total probability that any three of the $20 + 30$ flush valves and flush tanks are in simultaneous use is the sum of the above convolutions:

$$P(3) \approx 0.2394 \approx 23.94\%$$

Returning briefly to the issue of water demand, compare each convolution $P_v(x_v) \cdot P_t(j - x_v)$ to see the figure 0.1745 dominating its siblings, thus $(0, 3)$ is the most likely configuration in accordance with our estimate of $j^* = 3$.

We still need to establish that $P(3)$ is greater than all other $P(j)$. To study the $j = 2$ case, we need

$$P(2) = \sum_{x_v=0}^{2} P_v(x_v) \cdot P_t(j - x_v) \ ,$$

or:

$$P(2) = P_v(0) \cdot P_t(2) + P_v(1) \cdot P_t(1) + P_v(2) \cdot P_t(0)$$

Using the figures calculated above, we learn

$$P(2) \approx 0.1683 + 0.03181 + 0.001381 \approx 0.2015 = 20.15\% \, .$$

The $j = 1$ case is also done easily, as

$$P(1) = P_v(0) \cdot P_t(1) + P_v(1) \cdot P_t(0)$$

readily computes to

$$P(1) \approx 0.1044 + 0.009542 \approx 0.1139 \approx 11.39\% \, .$$

The $j = 0$ case is trivial from the information on hand, coming out to

$$P(0) \approx (0.7391)(0.04239) \approx 0.03133 \approx 3.133\% \, .$$

Reading this result backward, we see there is a 96.87% chance that at least one device is operating at a given moment.

Technically, we need to also check all additional $P(j)$ up to $j = 50$. This is best left to a machine, so we'll do one more case by hand, namely $j = 4$. For this, we need

$$P(4) = P_v(0) \cdot P_t(4) + P_v(1) \cdot P_t(3) + P_v(2) \cdot P_t(2) + P_v(3) \cdot P_t(1) + P_v(4) \cdot P_t(0) \ .$$

Most of these figures were calculated above, with the new members being:

$$P_v(4) = \binom{20}{4}(1-0.015)^{16}(0.015)^4 \approx 0.0001926$$
$$P_t(4) = \binom{30}{4}(1-0.1)^{26}(0.1)^4 \approx 0.1771$$

Turning the crank, one finds

$$P(4) = 0.1309 + 0.05315 + 0.007416 + 0.0004205 + .000008164 \, ,$$

or

$$P(4) \approx 0.1919 \approx 19.19\% \, .$$

To summarize, we found

$$P(0) = 3.133\%$$
$$P(1) = 11.39\%$$
$$P(2) = 20.15\%$$
$$P(3) = 23.94\%$$
$$P(4) = 19.19\% \, ,$$

which is maximal at $j = 3$. Given that $P(4)$ begins a downward trend, we can be sure that all subsequent $P(j > 4)$ are all less than $P(3)$.

Summing each percentage above, we conclude that there is a 77.80% chance that any number from zero to four devices are in use simultaneously. This means there is a 22.20% chance that any number $5 \leq j \leq 50$ devices are in use simultaneously.

Reading the trend in the $P(j)$, we estimate that the probability should be less than 1% by say, $j = 10$, thus we define a variable

$$k = 10 \approx 3j^* \approx 1 + 9 \, ,$$

which corresponds to one flush valve and nine flush tanks.

For a final flow rate we find

$$D(1, 9) = 1.04\bar{4} \, \frac{\text{gal}}{\text{s}} \, ,$$

which is about 63 gallons per minute.

Design Factor

To reiterate the last step, one reasons that $P(k) = 1\%$ corresponds to the worst probable use case. The special value k is also called a *design factor*. For this problem, there is about a 1% chance that more than 10 of the 50 fixtures are operating simultaneously.

1.3 Hunter's Calculation

While adequate, the above calculation is admittedly too detailed for application in the field, especially when there are multiple types of plumbing devices in the system.

To work toward something simpler, separately consider a (i) flush valve, (ii) flush tank, (iii) bathtub having the following characteristics[2]:

- The flush valve conveys 4 gallons over an interval of 9 seconds per use, and is used once every 5 minutes (300 s, twelve uses per hour).
- The flush tank conveys 4 gallons over an interval of 60 seconds per use, and is used once every 5 minutes (300 s, twelve uses per hour).
- A bathtub requires 16 gallons over an interval of 120 seconds, and is used once every 30 minutes (1800 s).

From these, we find that the flush valve operates a total of 108 seconds for every 3600. Thus the probability of any given flush valve being in use is

$$p_v = \frac{108}{3600} = 0.03 \,.$$

Similarly, the flush tank has

$$p_f = \frac{720}{3600} = 0.2 \,,$$

and finally for the bathtub:

$$p_b = \frac{120}{1800} = 0.06667$$

Then, one can immediately write the probability that x_v flush valves are in use out of V total valves:

$$P_v(x_v, V) = \binom{V}{x_v} (1 - p_v)^{V - x_v} \, p_v^{x_v}$$

From each value V we can derive a most-likely number of valves x_v^* in simultaneous use, along with a design factor k_v such that $P(k_v, V) = 1\%$. The very same can be said for flush tanks by switching indices $v \to t$, $V \to T$, leading to

$$P_t(x_t, T) = \binom{T}{x_t} (1 - p_t)^{V - x_t} \, p_t^{x_t} \,,$$

and then switching indices to b, B for bathtubs, we have

$$P_b(x_b, B) = \binom{B}{x_b} (1 - p_b)^{V - x_b} \, p_b^{x_b} \,.$$

Now we must find a design factor for each probability considered. To proceed, choose $V = T = B = 25$ and use a computer to find:

- $P_v(x_v, 25)$ equals 1% at $k_v = 3.671$
- $P_t(x_t, 25)$ equals 1% at $k_t = 0.4622$
- $P_b(x_b, 25)$ equals 1% at $k_b = 5.412$

Using each k-value, calculate the total water demand for each case of 25 fixtures:

$$D_v(k_v) = 3.671 \left(\frac{4\text{ gal}}{9\text{ s}}\right) = 97.89 \frac{\text{gal}}{\text{min}}$$
$$D_t(k_v) = 10.16 \left(\frac{4\text{ gal}}{60\text{ s}}\right) = 40.64 \frac{\text{gal}}{\text{min}}$$
$$D_b(k_b) = 5.412 \left(\frac{8\text{ gal}}{60\text{ s}}\right) = 43.30 \frac{\text{gal}}{\text{min}}$$

Repeating for $V = T = B = 50$, find:

- $P_v(x_v, 50)$ equals 1% at $k_v = 5.194$
- $P_t(x_t, 50)$ equals 1% at $k_t = 16.72$
- $P_b(x_b, 50)$ equals 1% at $k_b = 8.139$

Then, for 50 fixtures:

$$D_v(k_v) = 5.194 \left(\frac{4\text{ gal}}{9\text{ s}}\right) = 138.5 \frac{\text{gal}}{\text{min}}$$
$$D_t(k_v) = 16.72 \left(\frac{4\text{ gal}}{60\text{ s}}\right) = 66.88 \frac{\text{gal}}{\text{min}}$$
$$D_b(k_b) = 8.139 \left(\frac{8\text{ gal}}{60\text{ s}}\right) = 65.11 \frac{\text{gal}}{\text{min}}$$

Repeating again for $V = T = B = 75$, find

- $P_v(x_v, 75)$ equals 1% at $k_v = 6.509$
- $P_v(x_t, 75)$ equals 1% at $k_t = 22.84$
- $P_b(x_b, 75)$ equals 1% at $k_b = 10.58$

[2] National Bureau of Standards Report: BMS 65 by Late Dr. R. B. Hunter (1940)

Then, for 75 fixtures:

$$D_v(k_v) = 6.509\left(\frac{4\,\mathrm{gal}}{9\,\mathrm{s}}\right) = 173.6\,\frac{\mathrm{gal}}{\mathrm{min}}$$
$$D_t(k_v) = 22.84\left(\frac{4\,\mathrm{gal}}{60\,\mathrm{s}}\right) = 91.36\,\frac{\mathrm{gal}}{\mathrm{min}}$$
$$D_b(k_b) = 10.58\left(\frac{8\,\mathrm{gal}}{60\,\mathrm{s}}\right) = 84.64\,\frac{\mathrm{gal}}{\mathrm{min}}$$

Repeating once more for $V = T = B = 100$, find

- $P_v(x_v, 100)$ equals 1% at $k_v = 7.720$
- $P_v(x_t, 100)$ equals 1% at $k_t = 28.74$
- $P_b(x_b, 100)$ equals 1% at $k_b = 12.87$

Then, for 100 fixtures:

$$D_v(k_v) = 7.720\left(\frac{4\,\mathrm{gal}}{9\,\mathrm{s}}\right) = 205.9\,\frac{\mathrm{gal}}{\mathrm{min}}$$
$$D_t(k_v) = 28.74\left(\frac{4\,\mathrm{gal}}{60\,\mathrm{s}}\right) = 115.0\,\frac{\mathrm{gal}}{\mathrm{min}}$$
$$D_b(k_b) = 12.87\left(\frac{8\,\mathrm{gal}}{60\,\mathrm{s}}\right) = 103.0\,\frac{\mathrm{gal}}{\mathrm{min}}$$

Tables

It would be more efficient to state the above results and all subsequent calculations in table form. In the following, the first column holds the number of fixtures, and the remaining three columns hold the gallons-per-minute (gpm) flow rates through the respective fixtures:

Fixture (count)	**Valve** (gpm)	**Tank** (gpm)	**Bath** (gpm)
5	51.65	15.25	20.15
10	66.29	22.58	27.20
15	78.06	29.00	33.08
20	88.41	34.97	38.37
25	97.89	40.64	43.30
50	138.5	68.88	65.11
75	173.6	91.36	84.64
100	205.9	114.9	103.0
125	236.4	138.0	120.5
150	265.8	160.6	137.6
175	294.2	183.0	154.4
200	322.0	205.1	170.8
250	375.8	248.8	202.9
300	428.0	292.1	234.4
350	489.0	334.9	265.4

Fixture Units

One can construct a table similar to the above with a fixed gpm rate and a variable number of fixtures. Aiming for 50 gal/m, we find (i) $V = 5$ yields $D_v = 51.65$ gal/m, (ii) $T = 34$ yields $D_t = 50.42$ gal/m, (iii) $B = 33$ yields $D_v = 50.68$ gal/m. Together, we jot the ratio 5 : 34 : 33 for flush valves, flush tanks, and bathtubs, respectively. Repeating this for incrementing gpm rates yields the following:

Demand (gpm)	**Valve** (count)	**Tank** (count)	**Bath** (count)
50	5	34	33
100	27	85	96
150	58	139	169
200	96	195	246
250	137	252	326
300	181	310	407

The relationship between the flow rate in gallons per minute and total fixture count is more-or-less linear for each fixture type in the domain 150 gpm to 300 gpm. In ratio form, a subset of the above table reads:

Demand (gpm)	**Valve** (ratio)	**Tank** (ratio)	**Bath** (ratio)
150	1	2.397	2.914
200	1	2.031	2.563
250	1	1.839	2.380
300	1	1.713	2.249
(Average:)	(1)	(1.995)	(2.523)

Reading the bottom row, we reason that it would require 10 flush valves to meet the same gpm demand as 5 flush tanks. Similarly, the same 10 flush valves are equivalent to 4 bathtubs.

The numbers 10, 5, 4 are the respective *fixture units*, abbreviated FU, for the flush valve, flush tank, and bathtub, respectively.

Better Definition

We have proven that **a fixture unit is a dimensionless number representing any fixture's volumetric through-rate and probability of use. The total gpm demand of multiple fixtures is not the sum gpm demand of each fixture. Rather, one adds fixture units and uses Hunter's curve (or equivalent chart) to determine gpm demand in branches or the whole system.**

For a minimal example, we have that 10 fixture units corresponds to one flush valve, which to remind, conveys 4 gallons over an interval of 9 seconds of water per use, and is used once every 5 minutes. Note these numbers are decades old and have been refined since the 1940s.

Forensics

Reviewing the definitions that brought us here, it appears that only *International Plumbing Code, ICC A117.1-2017* is remotely coherent.

Meanwhile, the allusion to 'some arbitraryily chosen scale' in *Uniform Plumbing Code, IAMPO/ANSI 1 - 2021* should be absolutely clear at this point.

What are we to do with the definition listed in CMR 10.00? Back to flush valves... The modern flush valve, according to CMR 10.00, is classified as *Toilet, Valve Operated*, assigned to have 6 fixture units. Accordingly, such a modern valve conveys ≈ 1.5 gallons over 6 seconds. Thus we deduce that one fixture unit corresponds to 0.25 gallons conveyed over one second, or 15 gallons per minute. Up to a factor of ≈ 2, this reproduces CMR 10.00's definition.

Note, of course, that the 1.25 inch diameter pipe mentioned in CMR 10.00 never entered the analysis, and is nothing more than a distraction in the definition.

1.4 Hunter's Curve

Fixture unit calculations were originally carried out by Roy B. Hunter in the years leading to 1940 and published by the *National Bureau of Standards Report: BMS 65.*

Using 10, 5, 4 as scale factors, recast the 'gpm vs. count' table by multiplying all V by 10, all T by 5, and all B by 4. This produces an equivalent table with fixture counts replaced by fixture units:

Demand (gpm)	**Valve** (FU)	**Tank** (FU)	**Bath** (FU)
150	580	695	676
200	960	975	984
250	1370	1260	1304
300	1810	1550	1628

Extending the table above and plotting the information on a graph, much as Hunter did, leads to Figure 1.1. Using the Figure, Hunter reasoned:

'...the error made by using curve 2 for both flush tanks and bathtubs for any number of either up to 300 would be small. Also, the demand load relative to the number of fixture units may be approximately represented in this range by a smoother curve drawn above the two probability curves and merged with curve 1 as shown by the broken line in [the] Figure...'

That is, curve 2 and curve 3 are essentially interchangeable before the broken line, and then the broken line takes over for curves 2 and 3 until joining curve 1, giving rise to Figure 1.2. The result is called called *Hunter's Curve.*

Demand vs Fixture Units

Finally, we summarize the information in Hunter's curve using the tables that follow.

Demand (Load) (FU)	**Valve** (gpm)	**Tank** (gpm)
10	32.82	9.63
20	39.09	13.56
30	43.92	18.49*
40	48.02	27.20
50	51.65	30.24
60	54.97	33.08
70	58.04	35.78
80	60.93	38.37
90	63.67	40.87

Before the asterisk we derive values from curve 2. At the asterisk we use the average of curves 2 and 3. After the asterisk we use curve 3 and throughout the next table.

Demand (Load) (FU)	**Valve** (gpm)	**Tank** (gpm)
100	66.29	43.30
140	75.84	52.45
180	84.40	61.00
200	88.41	65.11
250	97.89	75.05
300	106.8	84.60
400	123.2	102.9
500	138.5	120.5
750	173.6	162.6

After ≈ 1000 fixture units, all values are represented by curve 1.

Demand (Load) (FU)	**Valve** (gpm)	**Tank** (gpm)
1000	205.9	205.1
1250	236.4	236.4
1500	265.8	265.8
1750	294.2	294.2
2000	322.0	322.0
2500	375.8	375.8
3000	428.0	428.0
4000	529.0	529.0
5000	626.8	626.8

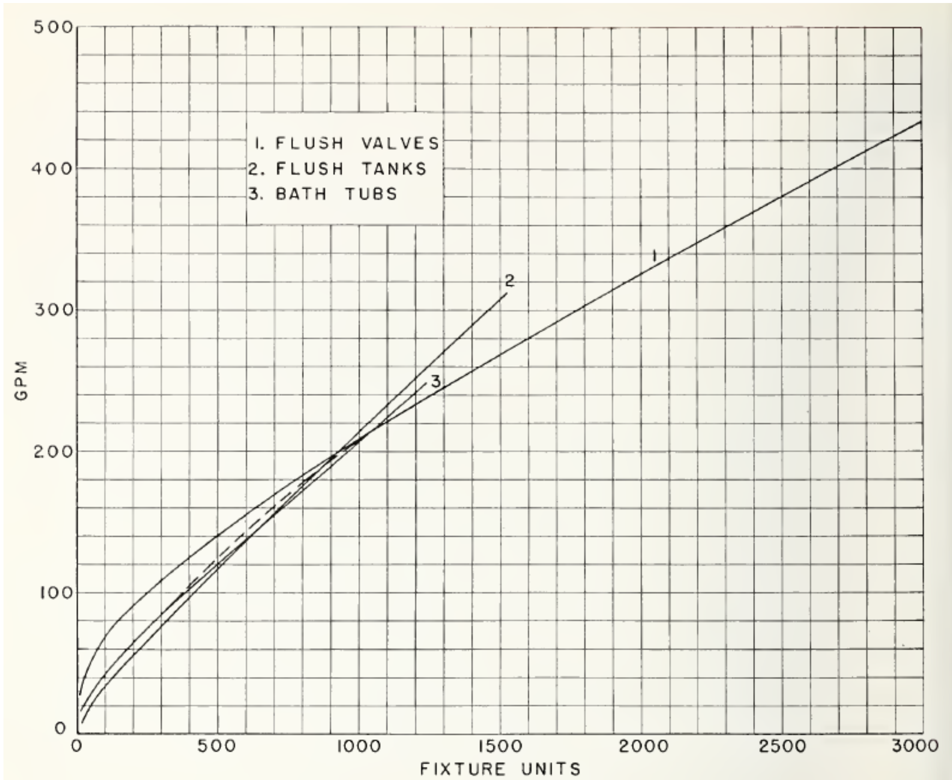

Figure 1.1: Gallons per minute versus fixture unit count for flush valves, flush tanks, and bathtubs. (*National Bureau of Standards Report: BMS 65*, 1940)

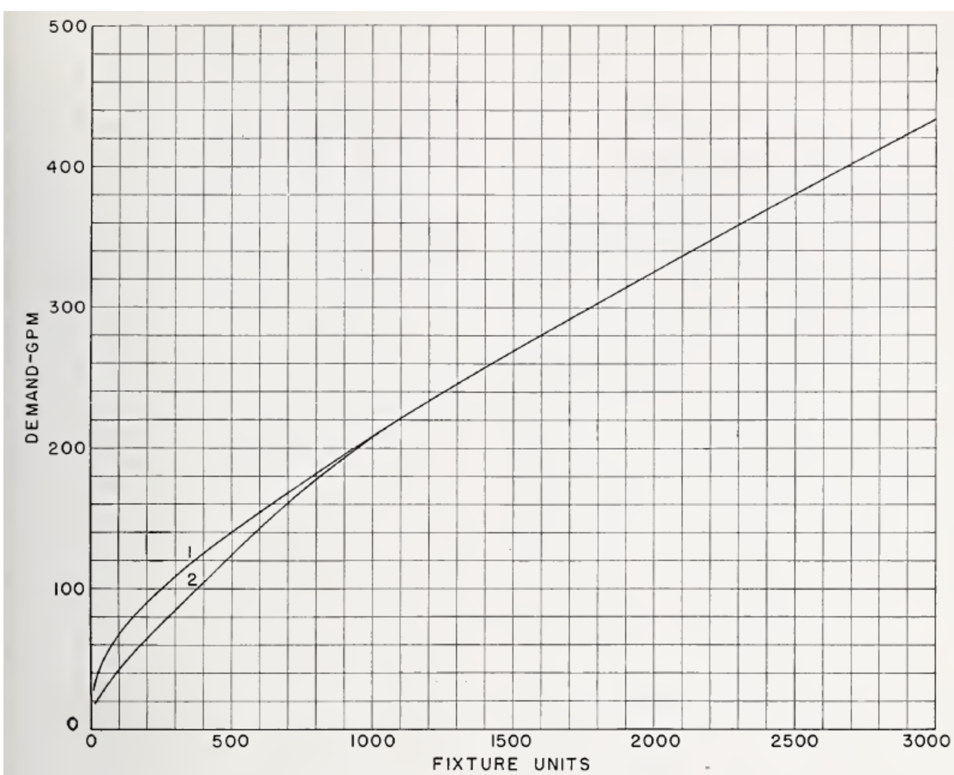

Figure 1.2: Hunter's Curve. (*National Bureau of Standards Report: BMS 65*, 1940)

2 Gas Pipe Sizing

2.1 Pipe Sizing Equations

In Revision 12/8/23 of CMR 10.00, the term 'gas' occurs 165 times. The term 'pipe' occurs 327 times. The combination 'gas pipe' occurs 0 times. Needless to say, 'gas pipe sizing' is a far cry from being as accessible to the learner as compared to other plumbing topics. (You can learn the five parts of a P-trap for free.)

Instead, one's first impression of gas pipe sizing may come from *ANSI Z223.1-23*, Chapter 6. In this, the reader is alerted to three methods for gas pipe sizing, providing means to choose the diameter of the gas pipe being used. The three methods are called (i) longest length method, (ii) branch method, (iii) hybrid pressure method. Each heuristic varies slightly from the others and each may return different answer for the same input.

Low Pressure Gas Formula

At the heart of gas pipe sizing lie a pair of equations that can (and will) be motivated from Newton's laws and Bernoulli's principle. Appearing as item `6.4.1` is the *low pressure gas formula*, which describes gases below a pressure of 1.5 psi, or 10.3 kPa:

$$D = \frac{Q^{0.381}}{19.17\left(\Delta H / \left(Cr \cdot L\right)\right)^{0.206}}$$

In the above: (i) D is the inner diameter of the pipe measured in inches. (ii) Q is the volumetric flow rate through a pipe measured in cubic feet per hour in standard conditions. (iii) ΔH is a pressure drop measured in inches of water column. (This is a confusing unit that we'll have to take on below.) (iv) L is the length of pipe measured in feet.

The terms Cr, and as we'll also see, Y are uncolorfully called *formula factors*. Natural gas has $Cr = 0.6094$ and $Y = 0.9992$. Undiluted propane has $Cr = 1.2462$ and $Y = 0.9910$.

High Pressure Gas Formula

A variation on the low pressure gas formula is the *high pressure gas formula*, valid for pressures higher than pressure of 1.5 psi, or 10.3 kPa:

$$D = \frac{Q^{0.381}}{18.93\left(\left(P_1^2 - P_2^2\right) Y / \left(Cr \cdot L\right)\right)^{0.206}}$$

The terms P_1, P_2 are the respective upstream and downstream pressures, measured in psia.

2.2 Darcy-Weisbach Equation

A fluid is *incompressible* if its mass per unit volume, also known as density ρ, is constant. As a general rule one can rely on all liquids being incompressible, while all gases are compressible. While there is never a reason to treat liquid as compressible, sometimes it is appropriate to treat gas as incompressible fluid.

Hydraulic Diameter

In the following we consider the motion of incompressible fluid through a straight pipe with cross sectional area A. For a notion of pipe diameter, define the *hydraulic diameter* D_H as

$$D_H = \frac{4A}{P} ,$$

where P is the perimeter of the pipe. For the case of cylindrical pipes, we have

$$D_H = \frac{4\pi D^2}{4\pi D} = D ,$$

which is just the circular diameter.

Volumetric Flow

Define the *wetted area* A_W as the portion of the cross-sectional area that is in contact with the moving fluid, i.e. the water flowing on the bottom of a pipe. The *volumetric flow*, denoted Q, is defined as

$$Q = A_W \langle v \rangle ,$$

where $\langle v \rangle = v$ is the mean velocity of the fluid.

Frictional Resistance

A model for the force of friction imposed on a moving fluid is attributed to Froude, providing the formula

$$F_R = f' \times A_W \times v^2 .$$

The coefficient f' is the frictional force per unit area with units to balance the v^2-term. We take A_W as the surface of pipe that is in contact with the fluid, namely $\pi D L$, where L is the length of pipe.

Bernoulli's Principle

A handy notion that we understand in terms of Newtonian laws and energy conservation is the famed *Bernoulli's principle*, stating that *for an incompressible fluid, the sum of its potential energy, pressure, and velocity remains constant.*

In the context of fluid moving through a pipe, one can expect that the effect of friction causes a loss of pressure ΔP between two given points in the pipe. Generally, one may write

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = \Delta P + P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \,,$$

which accounts for the kinetic, gravitational potential, and pressure terms responsible for the motion of the fluid from point '1' to point '2'.

Head Loss

A quantity

$$H_f = \frac{\Delta P}{\rho g} \,,$$

called the *head loss* is used to characterize the drop in pressure. Dimensionally, we have

$$[H_f] = \frac{\text{kg m}}{\text{s}^2\,\text{m}^2}\frac{\text{m}^3\,\text{s}^2}{\text{kg m}} = \text{m} \,,$$

i.e. pressure is interpreted as length (of water displaced in a column, for instance).

Darcy-Weisbach Equation

The effect of friction on a fluid moving in a pipe is characterized by the dimensionless *Darcy friction factor*, denoted f. To motivate this, consider the uniform motion of fluid at velocity v through a circular pipe of cross-sectional area A. With zero net acceleration, we use Newton's second law to establish

$$A\Delta P - F_R = 0 \,,$$

or after simplifying,

$$H_f = \frac{f'}{\rho g}\frac{4}{D}Lv^2 \,.$$

The combination f'/ρ is dimensionless, which we take as proportional to the coefficient of friction for fluid moving through the pipe. To keep consistent with common literature on the topic, define

$$f = \frac{8f'}{\rho}$$

so the above is finally written

$$H_f = f\frac{L}{D}\frac{v^2}{2g} \,,$$

known as the Darcy-Weisbach equation.

In terms of volumetric flow, the same equation is written

$$H_f = f\frac{8L}{\pi^2 g}\frac{Q^2}{D^5} \,.$$

If solving for the diameter of pipe involved, one has:

$$D = \left(\frac{f}{H_f}\frac{8L}{\pi^2 g}\right)^{1/5} Q^{2/5}$$

Laminar Flow

For the case of laminar flow, it can be shown that the friction factor f is exactly

$$f = \frac{64}{\text{Re}} \,,$$

where Re is the *Reynold's number*, defined as

$$\text{Re} = \frac{\rho D}{\mu}\langle v\rangle \,,$$

where μ is the viscosity of the fluid.