Plumbing Theory
MANUSCRIPT

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[1]

March 16, 2025

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Chapter 1

Plumbing Theory

Plumbing theory refers to any nonphysical aspect of plumbing. Any time spent in the classroom, looking at blueprints, estimating developed length, sketching pipes on a napkin, etc. - falls under plumbing theory.

Plumbers in working Massachusetts learn plumbing theory predominantly from the State *Plumbing Code* to ensure the safe and adequate operation of finished plumbing systems. Formally called *248 CMR 10.00*, the *Uniform State Plumbing Code* is available via *mass.gov* as PDF. Edition 12/8/23 is used for the majority of this study.

This study does not attempt to supplant or displace the myriad resources available to student and established plumbers. Nor does it attempt to convey any baseline understanding of any plumbing principle. For this, it is only appropriate to begin with Newton's laws and a study of thermodynamics and fluids.

Instead, the purpose here is to articulate certain parts of the standard student/aprentice literature that are lacking in adequate detail or lacking in explanation altogether.

1 Fixture Units

1.1 Review of Definitions

A term used heavily throughout the plumbing code is the *fixture unit*, which has a definition tailored for plumbers appearing in *Section 10.03: Definitions*:

Fixture Unit. *One cubic foot of water drained in a* 1.25 *inch pipe over a period of one minute. One cubic foot of water is equal to* 7.5 *gallons.*

The definition provided is alarmingly vague. Whether (or not) one cubic foot of water really drains through a 1.25 inch diameter pipe in precisely one minute is questionable. Needless to mention, one cubic foot is closer to 7.4813 gallons of liquid. As 7.5 has two digits of precision, the writer may as well have stated the pipe is 1.3 inches in diameter.

Dimensionality

Given the definition in CMR 10.03, one may expect that a fixture unit is a flow rate, perhaps measured in gallons per minute, and then perhaps multiplied (or divided?) by 1.25 somewhere, because surely the pipe diameter factors in. It turns out none of this is correct.

Browsing CMR 10.00 or comparable resources, one instead finds that fixture units are *not* measured in physical units, but are instead always dimensionless scale factors like 3, π, or 1000. This is also troublesome for the definition, because we lack three numbers to balance the units of gallons, minutes, and inches.

Alternate Definitions

Also found in *Section 10.03: Definitions* is the definition of *load factor*, which reads:

Load Factor. *The percentage of the total connected fixture unit flow which is likely to occur at any point in the drainage system. It varies with the type of occupancy, the total flow unit above this point being considered, and with the probability factor of simultaneous use.*

This definition is a slight improvement over the previous one by alluding to likelihood of occurrence and type of occupancy.

Sadly, the terms 'probability' and 'load factor' appear nowhere else in CMR 10.00. There is, of course, the suspiciously-related term 'demand factor' used in CMR 10.14, but this term is not defined in 10.03. A similar comment applies to the term 'factor value'. That is, 'factor value' seems to be related, is used, but is never defined in CMR 10.03.

Reaching for the *Uniform Plumbing Code, IAMPO/ANSI 1 - 2021*, one finds a dignified but unhelpful word salad explaining the fixture unit:

Fixture Unit. *A quantity in terms of which the load-producing effects on the plumbing system of different kinds of plumbing fixtures are expressed on some arbitrarily chosen scale.*

We finally encounter a sensible definition of the fixture unit in the *International Plumbing Code, ICC A117.1-2017*, which reads:

Drainage Fixture Unit. *A measure of the probable discharge into the drainage system by various types of plumbing fixtures. The drainage fixture-unit value for a particular fixture depends on its volume rate of drainage discharge, on the time duration of a single drainage operation and on the average time between successive operations.*

One can speculate as to why the definition of the fixture unit varies so vividly per authority. Regardless of this, any literal understanding of the matter will remain elusive if plumbing codes are the only resource.

A set of flash cards produced by Momentrix Media offers the following comment on fixture units:

A fixture unit is a unit used to measure the rate of water flow, equal to one cubic foot of water (roughly 7.48 *gallons) per minute.*

Arguing that the fixture unit is *equal* to one (physical) cubic foot of water per (real-time) minute is almost as bad as what we find in CMR 10.00.

The Plan

We've racked up plenty of technical karma by slaughtering the definition of the fixture unit, and the intent here is not to solve the problems with the definition stated in CMR 10.00 or other publications. Instead, the plan is to take a first-principles approach using probability theory in the same way as done by the inventor of the fixture unit, Roy B. Hunter, in the years leading to 1940.

We start with a homemade exercise called the **Valve and Tank Problem**, which is informed by a mixture of real data and more-or-less made up numbers corresponding to likeliness of occurrence, type of occupancy, etc., of certain plumbing fixtures.

Next, we pursue **Hunter's Calculation** to once-and-for-all force an understanding of the fixture unit. Such an effort also yields the so-called *Hunter's curve*, which is the source of the various 'gallons per minute versus fixture unit' tables occurring in the myriad of plumbing resources, including CMR 10.00. **You can skip the gritty calculations if math isn't your thing.**

Fixture unit values tend to change over time and location, however the work that follows will readily generalize to suit any fixture parameters. Data contemporary to the 1940s is used to ensure we're on track with the original work of R. B. Hunter.

1.2 Valve and Tank Problem

Two devices, or *fixtures* in American plumbing systems are the (i) *flush valve*, and (ii) the *flush tank*. The flush valve conveys 4 gallons of water over a period of 9 seconds.[1] The flush tank conveys 4 gallons in 60 seconds. Assume that all flush valves and flush tanks are used six times per hour on average.

If a certain building has $V = 20$ flush valves and $T = 30$ flush tanks, use only the information provided to (i) estimate the combined number of devices j being used simultaneously. (ii) Calculate the probability $P(j)$ of any j occurring and check that the estimate for j is correct. (iii) Calculate the probability that zero devices are in use at a given moment. (iv) Determine the number k at which $P(k) \approx 1\%$. (v) Calculate the final water demand using k.

Per hour (3600 seconds), a flush valve is operating for an average of 54 seconds. Similarly, a flush tank operates for 360 seconds. At a given moment, there is a

$$p_v = \frac{54}{3600} = 0.015$$

probability that a given flush valve is operating, and a

$$p_t = \frac{360}{3600} = 0.1$$

probability that a flush tank is operating.

Modal Analysis

Let x_v denote the number of flush valves in use simultaneously, and let x_t be the number of flush tanks being used such that

$$j = x_v + x_t$$

at a given moment.

Modeling each fixture as a weighted coin, we know from the binomial distribution that the most probable values for x_v, x_t are approximately the respective modes

$$x_v^* = V p_v - (1 - p_v) \approx V p_v$$
$$x_t^* = T p_t - (1 - p_t) \approx T p_t \; ,$$

or

$$x_v^* = (20)(0.015) = 0.3$$
$$x_t^* = (30)(0.1) = 3 \; .$$

The convoluted mode j^* ought to be the sum of the individual modes

$$j^* \approx x_v^* + x_t^* \; ,$$

[1] Figures gathered from PDH Course M126, *Sizing Plumbing Water System.* A. Bhatia. 2020. *www.PDHonline.prg*

and using the numbers on hand, one finds

$$j^* \approx 3.3 \, .$$

That is, we expect about three devices to be conveying water at a given moment. The total mode is essentially dominated by x_t^*.

In terms of x_v, x_t, the water demand rate $D(x_v, x_t)$ is given by

$$D(x_v, x_t) = x_v \left(\frac{4\text{ gal}}{9\text{ s}}\right) + x_t \left(\frac{4\text{ gal}}{60\text{ s}}\right) \, .$$

Using $j = 3$, it follows that x_v can take on any value 0, 1, 2, 3. Thus:

$$D(0,3) = 0.2\text{ gal/s}$$
$$D(1,2) = \left(0.44\overline{4} + 0.13\overline{3}\right)\text{ gal/s} = 0.57\overline{7}\text{ gal/s}$$
$$D(2,1) = \left(0.88\overline{8} + 0.066\overline{6}\right)\text{ gal/s} = 0.95\overline{5}\text{ gal/s}$$
$$D(3,0) = 1.3\overline{3}\text{ gal/s}$$

To make use of the above information, recall from the values of x_v^*, x_t^* that $(0,3)$ (zero flush valves in use, three flush tanks in use) is the most likely configuration at a given moment, and the typical water demand is estimated at 0.2 gal/s, or 12 gallons per minute.

Of course, we don't want to design based on typical use. It's much better to anticipate the worst-probable case, which is not the worst *possible* case. (To design based on the worst possible case scenario is inefficient, costly, or worse.)

Probabilistic Analysis

The probability that there are x_v flush valves in use is given by the binomial distribution:

$$P_v(x_v) = \binom{V}{x_v} (1 - p_v)^{V - x_v} p_v^{x_v}$$

Also in terms of x_v, we write for the flush tank:

$$P_t(j - x_v) = \binom{T}{j - x_v} (1 - p_t)^{T - (j - x_v)} p_t^{j - x_v}$$

The total probability is the sum of convolutions of the two above distributions:

$$P(j) = \sum_{x_v = \max(0, j-T)}^{\min(j,V)} P_v(x_v) \cdot P_t(j - x_v)$$

Using $j = 3$, the above probability is

$$P(3) = \sum_{x_v=0}^{3} P_v(x_v) \cdot P_t(j - x_v) \; ,$$

where:

$$P_v(x_v) = \binom{20}{x_v} (1 - 0.015)^{20 - x_v} (0.015)^{x_v}$$
$$P_t(j - x_v) = \binom{30}{3 - x_v} (1 - 0.1)^{30 - (3 - x_v)} (0.1)^{3 - x_v}$$

Evaluating $P(3)$ is quite a chore. For $x_v = 0$, we have:

$$P_v(0) = \binom{20}{0} (1 - 0.015)^{20} (0.015)^0 \approx 0.7391$$
$$P_t(3) = \binom{30}{3} (1 - 0.1)^{27} (0.1)^3 \approx 0.2361$$
$$P_v(0) \cdot P_t(3) \approx 0.1745$$

Continuing for $x_v = 1$:

$$P_v(1) = \binom{20}{1} (1 - 0.015)^{19} (0.015)^1 \approx 0.2251$$
$$P_t(2) = \binom{30}{2} (1 - 0.1)^{28} (0.1)^2 \approx 0.2277$$
$$P_v(1) \cdot P_t(2) \approx 0.05126$$

Continuing for $x_v = 2$:

$$P_v(2) = \binom{20}{2} (1 - 0.015)^{18} (0.015)^2 \approx 0.03257$$
$$P_t(1) = \binom{30}{1} (1 - 0.1)^{29} (0.1)^1 \approx 0.1413$$
$$P_v(2) \cdot P_t(1) \approx 0.004602$$

Continuing for $x_v = 3$:

$$P_v(3) = \binom{20}{3} (1 - 0.015)^{17} (0.015)^3 \approx 0.002976$$
$$P_t(0) = \binom{30}{0} (1 - 0.1)^{30} (0.1)^0 \approx 0.04239$$
$$P_v(3) \cdot P_t(0) \approx 0.0001262$$

The total probability that any three of the $20 + 30$ flush valves and flush tanks are in simultaneous use is the sum of the above convolutions:

$$P(3) \approx 0.2394 \approx 23.94\%$$

Returning briefly to the issue of water demand, compare each convolution $P_v(x_v) \cdot P_t(j - x_v)$ to see the figure 0.1745 dominating its siblings, thus $(0,3)$ is the most likely configuration in accordance with our estimate of $j^* = 3$.

We still need to establish that $P(3)$ is greater than all other $P(j)$. To study the $j = 2$ case, we need

$$P(2) = \sum_{x_v=0}^{2} P_v(x_v) \cdot P_t(j - x_v) \; ,$$

or:

$$P(2) = P_v(0) \cdot P_t(2) + P_v(1) \cdot P_t(1) + P_v(2) \cdot P_t(0)$$

Using the figures calculated above, we learn

$$P(2) \approx 0.1683 + 0.03181 + 0.001381 \approx 0.2015 = 20.15\% \; .$$

The $j = 1$ case is also done easily, as

$$P(1) = P_v(0) \cdot P_t(1) + P_v(1) \cdot P_t(0)$$

readily computes to

$$P(1) \approx 0.1044 + 0.009542 \approx 0.1139 \approx 11.39\% \; .$$

The $j = 0$ case is trivial from the information on hand, coming out to

$$P(0) \approx (0.7391)(0.04239) \approx 0.03133 \approx 3.133\% \; .$$

Reading this result backward, we see there is a 96.87% chance that at least one device is operating at a given moment.

Technically, we need to also check all additional $P(j)$ up to $j = 50$. This is best left to a machine, so we'll do one more case by hand, namely $j = 4$. For this, we need

$$P(4) = P_v(0) \cdot P_t(4) + P_v(1) \cdot P_t(3) + P_v(2) \cdot P_t(2) + P_v(3) \cdot P_t(1) + P_v(4) \cdot P_t(0) \; .$$

Most of these figures were calculated above, with the new members being:

$$P_v(4) = \binom{20}{4} (1 - 0.015)^{16} (0.015)^4 \approx 0.0001926$$
$$P_t(4) = \binom{30}{4} (1 - 0.1)^{26} (0.1)^4 \approx 0.1771$$

Turning the crank, one finds

$$P(4) = 0.1309 + 0.05315 + 0.007416 + 0.0004205 + .000008164 \; ,$$

or

$$P(4) \approx 0.1919 \approx 19.19\% \; .$$

To summarize, we found

$$P(0) = 3.133\%$$
$$P(1) = 11.39\%$$
$$P(2) = 20.15\%$$
$$P(3) = 23.94\%$$
$$P(4) = 19.19\% \; ,$$

which is maximal at $j = 3$. Given that $P(4)$ begins a downward trend, we can be sure that all subsequent $P(j > 4)$ are all less than $P(3)$.

Summing each percentage above, we conclude that there is a 77.80% chance that any number from zero to four devices are in use simultaneously. This means there is a 22.20% chance that any number $5 \leq j \leq 50$ devices are in use simultaneously.

Reading the trend in the $P(j)$, we estimate that the probability should be less than 1% by say, $j = 10$, thus we define a variable

$$k = 10 \approx 3j^* \approx 1 + 9 \; ,$$

which corresponds to one flush valve and nine flush tanks.

For a final flow rate we find

$$D(1, 9) = 1.04\bar{4} \, \frac{\text{gal}}{\text{s}} \; ,$$

which is about 63 gallons per minute.

Design Factor

To reiterate the last step, one reasons that $P(k) = 1\%$ corresponds to the worst probable use case. The special value k is also called a *design factor*. For this problem, there is about a 1% chance that more than 10 of the 50 fixtures are operating simultaneously.

1.3 Hunter's Calculation

While adequate, the above calculation is admittedly too detailed for application in the field, especially when there are multiple types of plumbing devices in the system.

To work toward something simpler, separately consider a (i) flush valve, (ii) flush tank, (iii) bathtub having the following characteristics[2]:

- The flush valve conveys 4 gallons over an interval of 9 seconds per use, and is used once every 5 minutes (300 s, twelve uses per hour).
- The flush tank conveys 4 gallons over an interval of 60 seconds per use, and is used once every 5 minutes (300 s, twelve uses per hour).

[2] National Bureau of Standards Report: BMS 65 by Late Dr. R. B. Hunter (1940)

- A bathtub requires 16 gallons over an interval of 120 seconds, and is used once every 30 minutes (1800 s).

From these, we find that the flush valve operates a total of 108 seconds for every 3600. Thus the probability of any given flush valve being in use is

$$p_v = \frac{108}{3600} = 0.03 \,.$$

Similarly, the flush tank has

$$p_f = \frac{720}{3600} = 0.2 \,,$$

and finally for the bathtub:

$$p_b = \frac{120}{1800} = 0.06667$$

Then, one can immediately write the probability that x_v flush valves are in use out of V total valves:

$$P_v\left(x_v, V\right) = \binom{V}{x_v} \left(1 - p_v\right)^{V - x_v} p_v^{x_v}$$

From each value V we can derive a most-likely number of valves x_v^* in simultaneous use, along with a design factor k_v such that $P\left(k_v, V\right) = 1\%$. The very same can be said for flush tanks by switching indices $v \to t$, $V \to T$, leading to

$$P_t\left(x_t, T\right) = \binom{T}{x_t} \left(1 - p_t\right)^{V - x_t} p_t^{x_t} \,,$$

and then switching indices to b, B for bathtubs, we have

$$P_b\left(x_b, B\right) = \binom{B}{x_b} \left(1 - p_b\right)^{V - x_b} p_b^{x_b} \,.$$

Now we must find a design factor for each probability considered. To proceed, choose $V = T = B = 25$ and use a computer to find:

- $P_v\left(x_v, 25\right)$ equals 1% at $k_v = 3.671$
- $P_t\left(x_t, 25\right)$ equals 1% at $k_t = 0.4622$
- $P_b\left(x_b, 25\right)$ equals 1% at $k_b = 5.412$

Using each k-value, calculate the total water demand for each case of 25 fixtures:

$$D_v\left(k_v\right) = 3.671 \left(\frac{4\text{ gal}}{9\text{ s}}\right) = 97.89 \frac{\text{gal}}{\text{min}}$$

$$D_t\left(k_v\right) = 10.16 \left(\frac{4\text{ gal}}{60\text{ s}}\right) = 40.64 \frac{\text{gal}}{\text{min}}$$

$$D_b\left(k_b\right) = 5.412 \left(\frac{8\text{ gal}}{60\text{ s}}\right) = 43.30 \frac{\text{gal}}{\text{min}}$$

Repeating for $V = T = B = 50$, find:

- $P_v\left(x_v, 50\right)$ equals 1% at $k_v = 5.194$
- $P_t\left(x_t, 50\right)$ equals 1% at $k_t = 16.72$
- $P_b\left(x_b, 50\right)$ equals 1% at $k_b = 8.139$

Then, for 50 fixtures:

$$D_v\left(k_v\right) = 5.194 \left(\frac{4\text{ gal}}{9\text{ s}}\right) = 138.5 \frac{\text{gal}}{\text{min}}$$

$$D_t\left(k_v\right) = 16.72 \left(\frac{4\text{ gal}}{60\text{ s}}\right) = 66.88 \frac{\text{gal}}{\text{min}}$$

$$D_b\left(k_b\right) = 8.139 \left(\frac{8\text{ gal}}{60\text{ s}}\right) = 65.11 \frac{\text{gal}}{\text{min}}$$

Repeating again for $V = T = B = 75$, find:

- $P_v\left(x_v, 75\right)$ equals 1% at $k_v = 6.509$
- $P_v\left(x_t, 75\right)$ equals 1% at $k_t = 22.84$
- $P_b\left(x_b, 75\right)$ equals 1% at $k_b = 10.58$

Then, for 75 fixtures:

$$D_v\left(k_v\right) = 6.509 \left(\frac{4\text{ gal}}{9\text{ s}}\right) = 173.6 \frac{\text{gal}}{\text{min}}$$

$$D_t\left(k_v\right) = 22.84 \left(\frac{4\text{ gal}}{60\text{ s}}\right) = 91.36 \frac{\text{gal}}{\text{min}}$$

$$D_b\left(k_b\right) = 10.58 \left(\frac{8\text{ gal}}{60\text{ s}}\right) = 84.64 \frac{\text{gal}}{\text{min}}$$

Repeating once more for $V = T = B = 100$, find:

- $P_v\left(x_v, 100\right)$ equals 1% at $k_v = 7.720$
- $P_v\left(x_t, 100\right)$ equals 1% at $k_t = 28.74$
- $P_b\left(x_b, 100\right)$ equals 1% at $k_b = 12.87$

Then, for 100 fixtures:

$$D_v\left(k_v\right) = 7.720 \left(\frac{4\text{ gal}}{9\text{ s}}\right) = 205.9 \frac{\text{gal}}{\text{min}}$$

$$D_t\left(k_v\right) = 28.74 \left(\frac{4\text{ gal}}{60\text{ s}}\right) = 115.0 \frac{\text{gal}}{\text{min}}$$

$$D_b\left(k_b\right) = 12.87 \left(\frac{8\text{ gal}}{60\text{ s}}\right) = 103.0 \frac{\text{gal}}{\text{min}}$$

1.4 Fixture Unit Defined

After suffering (or skipping) the above analysis, it would be efficient to restate all results in table form with assistance from a computer or programmable calculator to crunch the numbers.

Fixture Count v.s. GPM

In the following table, the first column holds a number of fixtures, and the remaining three columns hold the gallons-per-minute (gpm) flow rates through the respective fixtures. This is an extended summary of Hunter's calculation:

Fixture (count)	**Valve** (gpm)	**Tank** (gpm)	**Bath** (gpm)
5	51.65	15.25	20.15
10	66.29	22.58	27.20
15	78.06	29.00	33.08
20	88.41	34.97	38.37
25	97.89	40.64	43.30
50	138.5	68.88	65.11
75	173.6	91.36	84.64
100	205.9	114.9	103.0
125	236.4	138.0	120.5
150	265.8	160.6	137.6
175	294.2	183.0	154.4
200	322.0	205.1	170.8
250	375.8	248.8	202.9
300	428.0	292.1	234.4
350	489.0	334.9	265.4

GPM v.s. Fixture Count

One can derive a similar table with a fixed gpm rate and a variable number of fixtures. Aiming for 50 gal/m, we find (i) $V = 5$ yields $D_v = 51.65$ gal/m, (ii) $T = 34$ yields $D_t = 50.42$ gal/m, (iii) $B = 33$ yields $D_v = 50.68$ gal/m. Together, we jot the ratio $5 : 34 : 33$ for flush valves, flush tanks, and bathtubs, respectively. Repeating this for incrementing gpm rates yields the following:

Demand (gpm)	**Valve** (count)	**Tank** (count)	**Bath** (count)
50	5	34	33
100	27	85	96
150	58	139	169
200	96	195	246
250	137	252	326
300	181	310	407

The relationship between the flow rate in gallons per minute and total fixture count is more-or-less linear for each fixture type in the domain 150 gpm to 300 gpm, thus we will toss out any data outside this window.

GPM v.s. Flush Valve Ratio

The next move is to express all fixtures in terms of the flush valve as Hunter did. Going across each row, divide the respective numbers of flush tanks and bathtubs by the number of flush valves. Also divide the number of flush valves by itself to attain 1. Doing so yields:

Demand (gpm)	**Valve** (ratio)	**Tank** (ratio)	**Bath** (ratio)
150	1	2.397	2.914
200	1	2.031	2.563
250	1	1.839	2.380
300	1	1.713	2.249
(Average:)	(1)	(1.995)	(2.523)

Interpreting the bottom row, we reason that it would require 10 flush valves to meet the same gpm demand as about 5 flush tanks (because $1.995 \times 5 \approx 10$). Similarly, the same 10 flush valves are equivalent to 4 bathtubs (because $2.523 \times 4 \approx 10$).

The numbers 10, 5, 4 are the respective *fixture units*, abbreviated FU, for the flush valve, flush tank, and bathtub, respectively.

Better Definition

We have proven that **any plumbing fixture is characterized by one or more fixture units, a dimensionless number informed by the fixture's volume through-rate and probability/duration of use. The total gpm demand of multiple fixtures is not the sum gpm demand of each fixture. Rather, count the total fixture units and use Hunter's curve (or equivalent chart) to determine total gpm demand.**

Corollary

The fixture unit does *not* differentiate between water supply versus waste drainage. This is because in a typical building, the average water supply is equal to the average waste drainage (minus whatever passes through things like roof drains.)

To accomodate exceptions to the above, it's prudent to differentiate between water supply fixture units (WSFU) versus waste drainage fixture units (DFU). Fixture unit values tend to vary in each regime, as the conditions and contents of a given pipe may factor into that pipe's ability to pass volume through the system.

Plumbers have derived or inherited various prejudices against the universality of 'fixture unit' and have adopted their own terms. CMR 10.14 offers the term 'factor value' for water supply, however this term is not defined in CMR 10.03. Meanwhile, 'fixture unit' is seemingly reserved for waste and drainage.

Forensics

Reviewing the definitions that brought us here, it appears that *International Plumbing Code*, *ICC A117.1-2017* contains no inaccuracies, but also skimps elaboration on what 'kind' of measure to which the definition alludes.

Meanwhile, in *IAMPO/ANSI 1 - 2021*, *Uniform Plumbing Code*, the allusion to 'some arbitrarily chosen scale' in should be clear at this point. They mean Hunter's curve (See Figure 1.2).

What are we to do with the definition listed in CMR 10.00? Back to flush valves... The modern flush valve, according to CMR 10.15, is classified as *Toilet, Valve Operated*, assigned to have 6 fixture units. Accordingly, such a modern valve conveys ≈ 1.5 gallons over 6 seconds. Thus we deduce that one fixture unit corresponds to 0.25 gallons conveyed over one second, or 15 gallons per minute. Up to a factor of ≈ 2, this reproduces CMR 10.03's definition.

Note, of course, that the 1.25 inch diameter pipe mentioned in CMR 10.00 never entered the analysis, and is nothing more than a distraction.

1.5 Hunter's Curve

Fixture unit calculations were originally carried out by Roy B. Hunter in the years leading to 1940 and published by the *National Bureau of Standards Report: BMS 65.*

Using 10, 5, 4 as scale factors, recast the 'gpm vs. count' table by multiplying all V by 10, all T by 5, and all B by 4. This produces an equivalent table with fixture counts replaced by fixture units:

Demand (gpm)	**Valve** (FU)	**Tank** (FU)	**Bath** (FU)
150	580	695	676
200	960	975	984
250	1370	1260	1304
300	1810	1550	1628

Extending the table above and plotting the information on a graph, much as Hunter did, leads to Figure 1.1. Using the Figure, Hunter reasoned:

'...the error made by using curve 2 for both flush tanks and bathtubs for any number of either up to 300 would be small. Also, the demand load relative to the number of fixture units may be approximately represented in this range by a smoother curve drawn above the two probability curves and merged with curve 1 as shown by the broken line in [the] Figure...'

That is, curve 2 and curve 3 are essentially interchangeable before the broken line, and then the broken line takes over for curves 2 and 3 until joining curve 1, giving rise to Figure 1.2. The result is called called *Hunter's Curve.*

Demand vs Fixture Units

Finally, we summarize the information in Hunter's curve using the tables that follow.

Demand (Load) (FU)	**Valve** (gpm)	**Tank** (gpm)
10	32.82	9.63
20	39.09	13.56
30	43.92	18.49*
40	48.02	27.20
50	51.65	30.24
60	54.97	33.08
70	58.04	35.78
80	60.93	38.37
90	63.67	40.87

Before the asterisk we derive values from curve 2. At the asterisk we use the average of curves 2 and 3. After the asterisk we use curve 3 and throughout the next table.

Demand (Load) (FU)	**Valve** (gpm)	**Tank** (gpm)
100	66.29	43.30
140	75.84	52.45
180	84.40	61.00
200	88.41	65.11
250	97.89	75.05
300	106.8	84.60
400	123.2	102.9
500	138.5	120.5
750	173.6	162.6

After ≈ 1000 fixture units, all values are represented by curve 1.

Demand (Load) (FU)	**Valve** (gpm)	**Tank** (gpm)
1000	205.9	205.1
1250	236.4	236.4
1500	265.8	265.8
1750	294.2	294.2
2000	322.0	322.0
2500	375.8	375.8
3000	428.0	428.0
4000	529.0	529.0
5000	626.8	626.8

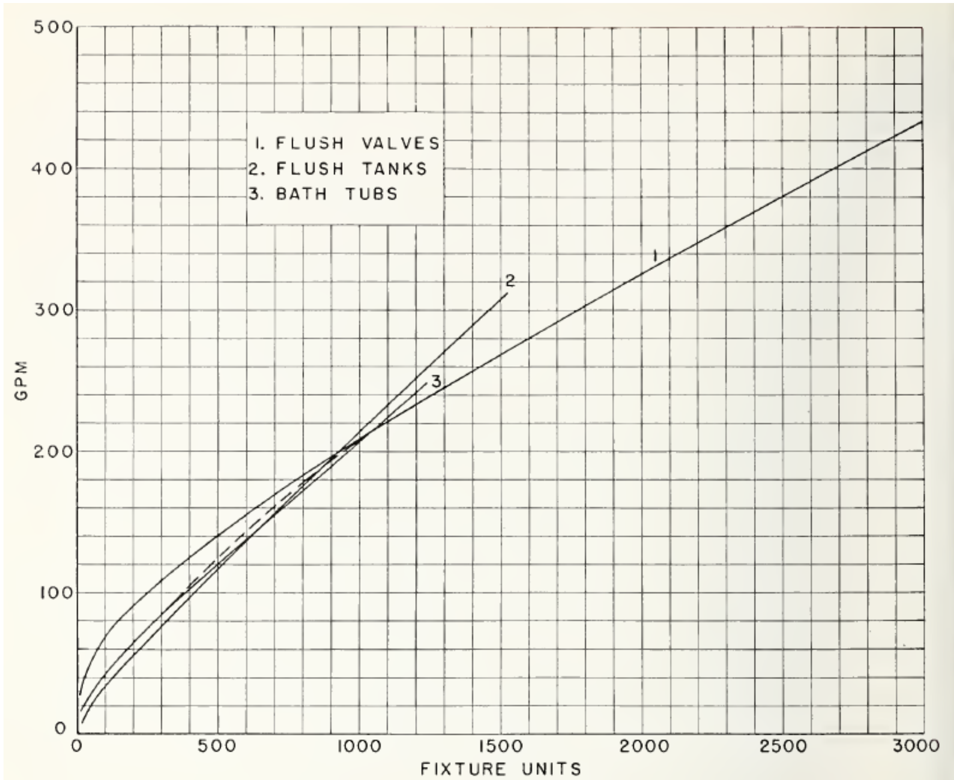


Figure 1.1: Gallons per minute versus fixture unit count for flush valves, flush tanks, and bathtubs. (*National Bureau of Standards Report: BMS 65*, 1940)

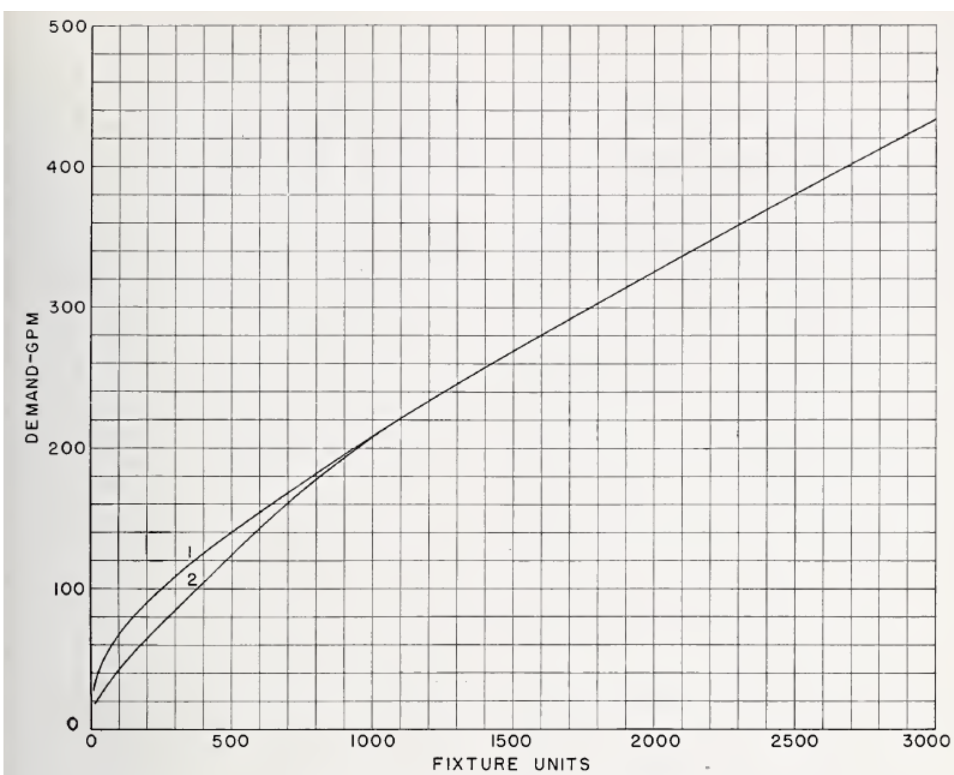


Figure 1.2: Hunter's Curve. (*National Bureau of Standards Report: BMS 65*, 1940)

1.6 Water Pipe Sizing (in MA)

According to CMR 10.14, the water supply fixture unit (WSFU) does not apply or exist. Instead, CMR 10.14.2(a) introduces three terms, namely 'factor value', 'demand factor', and 'capacity value', none of which are defined in CMR 10.03. Regardless, these together do the equivalent job of the water supply fixture unit.

Water Pipe Sizing

Figures 1.3, 1.4, 1.5 occur in succession in CMR 10.14 as Table 1, Table 2, Table 3, respectively. These essentially tell the story of sizing water supply pipe.

CMR 10.14 Table 1 lists common plumbing fixtures with the associated factor value. Between these is the nominal pipe size for the fixture. For multiple fixtures on the same branch, we're still adding factor values as if they're water supply fixture units. The total factor value for a branch or building is the sum of the individual factor values.

Next, CMR 10.14 Table 2 implies how 'demand factor' figures in. In particular, one must consider the type of 'occupancy use', and then multiply the total factor value by the demand factor, and this yields the capacity value:

$$(\text{Factor value}) \times (\text{Demand factor}) = \text{Capacity value}$$

One may think of capacity value as a context-adjusted water supply fixture unit. With the capacity value in hand, one turns to CMR 10.14 Table 3 to select the appropriate pipe diameter.

Type of Fixture or Device	Nominal Pipe Size (Inches)	Factor Value
Bathtub (with or without single shower head)	½	2
Bidet	⅜	1
Drinking Water Station	⅜	1
Dishwasher (Domestic)	½	2
Dishwasher (Commercial)	¾	6
Kitchen sink, Residential	½	2
Kitchen sink, Commercial (Pot and Scullery)	¾	6
Vegetable Prep or Bar Sink (Residential)	½	2
Hand Wash Sinks	⅜	1
Shampoo Sinks	⅜	1
Lavatory	⅜	1
Utility Laundry Sinks 1, 2, or 3 compartments	½	2
Shower Valve (single head)	½	2
Shower Valve (Multiple heads)	¾	6
Sinks (service, slop)	½	2
Sinks flushing rim	¾	6
Laundry Valve	½	2
Urinal (flushometer type)	¾	6
Toilet (tank type)	⅜	1
Toilet (flush valve type)	1	12
Hose Connections/Sillcocks/Wall Hydrants	½	2

Figure 1.3: CMR 10.14 Table 1: Minimum Sizes of Individual Fixture Branches and Factor Values.

Occupancy Use	Demand Factors
Residential	
One or Two Family Dwelling	0.50
Multi-residential	0.35
Hotel	0.70
School	
General	0.75
Shower Room	1.00
Institutional	
General	0.45
Assembly	
General	0.25
Restaurant, Café	0.70
Club House	0.60
Business and Mercantile	
General	
Industrial	0.25
Laundry	1.00
INDUSTRIAL	
General, Exclusive of Process Piping	0.90

Figure 1.4: CMR 10.14 Table 2: Occupancy and Demand Factor.

Nominal Pipe or Tubing Sizes (inches)	Capacity Value
½	1 to 4
¾	4.1 to 9
1	9.1 to 16.5
1¼	16.6 to 28
1½	28.1 to 55
2	55.1 to 107.5
2½	107.6 to 182.5
3	182.6 to 287.5
3½	287.6 to 425
4	425.1 to 700
5	700.1 to 1100
6	1100.1 to 1300

Figure 1.5: CMR 10.14 Table 3: Capacity Values for Service, Mains, Risers and/or Branches.

Capacity Value v.s. Diameter

The exact relationship between the pipe diameter and the capacity value is a bit elusive when looking at numbers alone. Since cross-sectional area grows as the square of the diameter, one could roughly assume that the capacity value versus pipe diameter follows a similar law. Trying this out, one could let x represent the diameter in inches and y represent the capacity value. Then, for some unknown coefficient A, we have

$$y = Ax^2 .$$

Trying out a few values, one finds:

$$A = \frac{(16.5 + 9.1)/2}{1^2} = 12.8$$
$$A = \frac{(107.5 + 55.1)/2}{2^2} = 20.33$$
$$A = \frac{(287.5 + 182.6)/2}{3^2} = 26.12$$
$$A = \frac{(700 + 425.1)/2}{4^2} = 35.16$$

These values for A are all over the place. The model $y = Ax^2$ doesn't work.

Going instead for a power law model, we write

$$y = Ax^B \ ,$$

where A and B are unknown coefficients. The y-variable still represents capacity value, and the x-variable still represents pipe diameter in inches. Continue by writing

$$\ln(y) = \ln(A) + B\ln(x) \ ,$$

and the problem is now fit for linear regression analysis. Sparing the details, one finds

$$A \approx 13.858$$
$$B \approx 2.5686$$

such that:

$$y = (13.858)\, x^{2.5686} \tag{1.1}$$

Up to a small error, Equation (1.1) completely encodes CMR 10.14 Table 3 and can be used in place of it.

Diameter v.s. Capacity Value

We can also derive a formula for the pipe diameter as a function of capacity value, which amounts to solving Equation (1.1) for x:

$$x = \left(\frac{y}{13.858}\right)^{1/2.5686} = (0.35935)\, y^{0.38932}$$

Of course, the result will come out to an exotic decimal. Treat any outputs as approximate and use CMR 10.14 Table 3 to inform any final decisions.

Volume Flow v.s. Diameter

Conspicuously absent from the water pipe sizing heuristic for plumbing in Massachusetts is any mention of the volume passing through any pipe of a given size.

Luckily, this problem has been studied thoroughly and charts and tables are freely available. Choosing one from *WCP Online*, we see in Figure 1.6 the relation between volume flow in gallons per minute versus pipe diameter at a 15 psi pressure drop across the pipe.

Table 1. Volumetric flows in pipes

Pipe (ID) inches	Inside area (sq. inches)	Linear flow (feet/sec) 10-20 psi ΔP	Volume flow (gpm) at 15 psi ΔP
0.25	0.05	3.0-4.3	0.45
0.375	0.11	4.0-6.0	1.35
0.50	0.20	5.0-7.5	3.0
0.75	0.44	6.0-10.0	8.9
1.0	0.78	7.0-11.5	19.3
1.25	1.23	9.0-13.2	38.3
1.5	1.77	10.0-14.6	66
2.0	3.14	13-18	123
3.0	7.07	16-24	362
4.0	12.56	19-28	775

Figure 1.6: Hydrodynamic design, part 2: Flows, pipe. (2013, February 3). WCP Online. https://wcponline.com/2013/02/03/hydrodynamic-design-part-2-flows-pipe/

Supposing that the capacity value in Figure 1.5 is related to the volume flow column in Figure 1.6, we can do a similar power law fit

$$y = Ax^B$$

on the first and last columns to learn

$$A \approx 19.495$$
$$B \approx 2.6963$$

such that:

$$y = (19.495)\, x^{2.6963} \tag{1.2}$$

Capacity Value v.s. Volume Flow

Paying attention to the exponents we're found, we see 2.5686 from Equation (1.1) is comparable to the the value 2.6963 from Equation (1.2). In parciular, consider the ratio of the two equations

$$R = \frac{(13.858)\, x^{2.5686}}{(19.495)\, x^{2.6963}} \approx (0.71085)\, x^{-0.1277}$$

The exponent on x is close enough to zero to ignore the whole x-term, leaving the ratio $R \approx 0.71085$. Interpreting this, we see that for a given pipe diameter, the capacity value (CMR 10.14 Table 3) is roughly equal to 0.71085 times the volume flow (Figure 1.6). Said another way, the volume flow in gallons per minute through a pipe is roughly (the reciprocal of 0.71085, thus) 1.4068 times the capacity value:

$$\text{Capacity value} \approx 0.71085 \times (\text{Volume flow})$$
$$\text{Volume flow} \approx 1.4068 \times (\text{Capacity value})$$

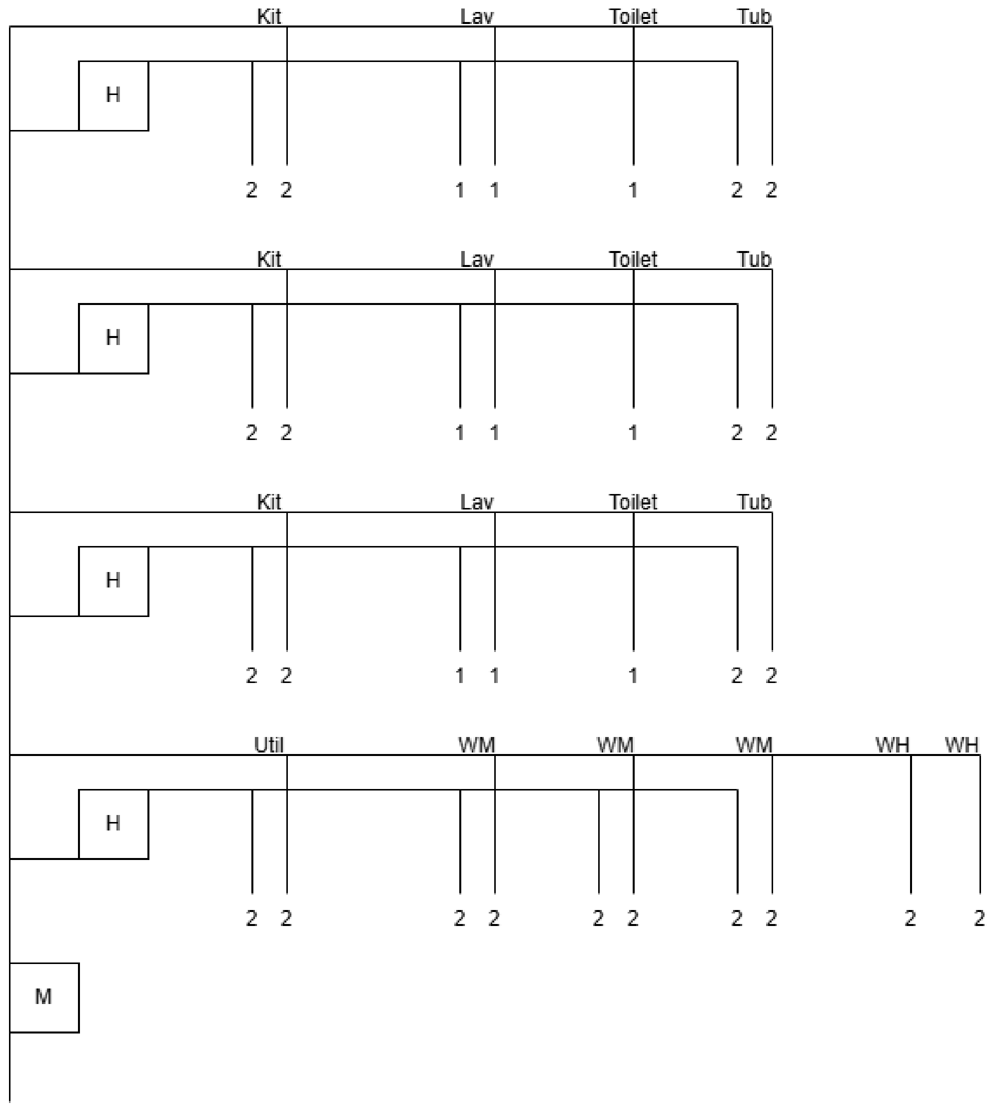


Figure 1.7: .

Single Dwelling

In a single dwelling, the capacity value for both hot and cold never exceeds 4, thus all horizontal lines in the sketch (single dwelling only) correspond to 1/2 inch diameter pipe.

Basement

Main to Meter

For the entire building, we sum the demand of three dwellings plus the basement to give $15+8=23$ units for hot and $18+12=30$ units for cold. The sum of these is $23+30=53$ units.

Multiplying each by the demand factor of 0.35, the hot water capacity value is $5.25+2.8=8.05$ and the cold water capacity value is $6.3+4.2 = 10.5$.

Single Dwelling					
Fixture	Nom. D. (in.)	Count	Hot FV	Cold FV	
Kit. Sink	0.5	1	2	2	
Lav	0.5	1	1	1	
Toilet	0.5	1	0	1	
Tub	0.5	1	2	2	
Subtotal FV			5	6	11
Capacity Value (×0.35)			1.75	2.1	3.85
Iterations (×3)					
Total FV			15	18	33
Capacity Value (×0.35)			5.25	6.3	11.55

Basement					
Fixture	Nom. D. (in.)	Count	Hot FV	Cold FV	
Util. Sink	0.5	1	2	2	
Wash. Mach.	0.5	3	6	6	
Wall Hyd.	0.5	2	0	4	
Total FV			8	12	20
Capacity Value (×0.35)			2.8	4.2	7.0

The total capacity value is the sum of these, or $8.05 + 10.5 = 18.55$.

Building				
Zone	Count	Hot FV	Cold FV	
Single Dwelling	3	15	18	
Basement	1	8	12	
Total FV		23	30	53
Capacity Value (×0.35)		8.05	10.5	18.55

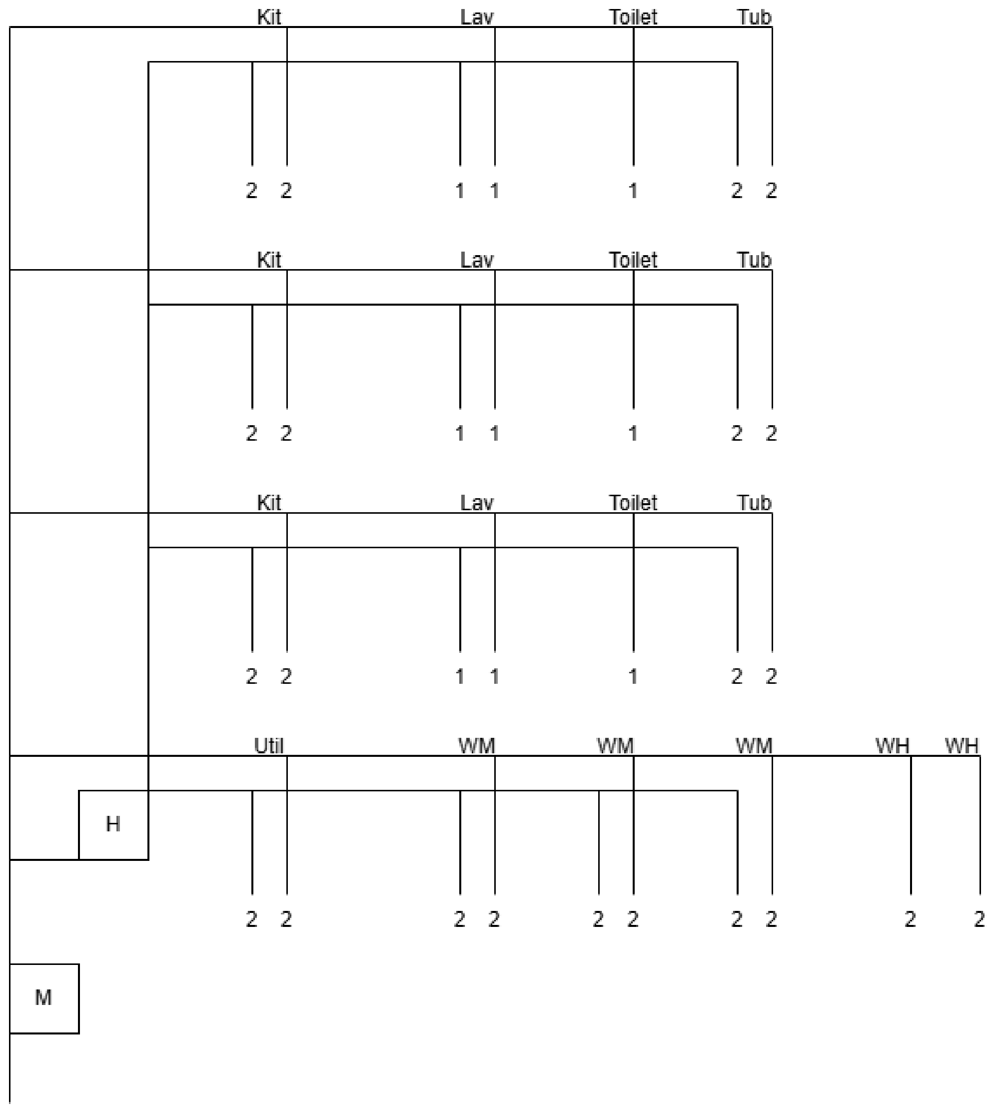


Figure 1.8: .

1.7 Waste Pipe Sizing (in MA)

Waste pipe sizing makes use of the term 'fixture unit' to classify the drainage from any given fixture. Figure 1.9 shows CMR 10.15 Table 1 for a variety of common fixtures.

Type of fixture or group of fixtures	Fixture Unit Value
Bathtub or Tub & Shower Unit	2
Bidet	2
Dental chair unit or cuspidor	1
Dental lavatory	1
Drinking fountain/Water Station	1
Dishwasher, commercial	6
Dishwasher, Residential	1
Floor/trench drain 2-inch	4
Floor/trench drain 3-inch	5
Floor/trench drain 4-inch	6
Kitchen sink Residential (with or without disposer)	2
Lavatory with 1-1/4" outlet	1
Laundry Connection Residential	3
Laundry/Utility Sink	2
Shower stall Residential	2
Showers (group) per head	2
Sinks:	
Surgeons	3
Flushing rim (with valve)	6
Service Sink with Trap Standard	3
Service Sink with P-Trap	2
Commercial Pot, scullery, *etc.* (each section) *See Note 1 Below*	4
Shampoo	2
Toilet, Tank Type	4
Toilet, Valve Operated	6
Urinal, pedestal, siphon jet blowout	6
Urinal, wall lip	4
Urinal, Waterless	1
Wash sink (circular or multiple) each 20 inches of usable length	1
Sizes for fixtures not listed above:	
1¼ inch or less	1
1½ inches	2
2 inches	3
2½ inches	4
3 inches	5
4 inches	6

Figure 1.9: CMR 10.15 Table 1: Fixture Unit Values for Various Plumbing Fixtures.

There are several notes (not explicitly listed here) to go along with the Table pertaining to grease interceptors, devices with (semi-)consinuous flow, and floor drains handling varying surface areas.

Waste Pipe Sizing

As per usual with fixture units, one must take the sum of all fixture units in a particular branch or building and sizing the pipe(s) accordingly.

There are four configurations for the slope of a pipe suggested in CMR 10.15. Measured from the horizontal, these are:

- 1/8 inch per foot (0.597°)
- 1/4 inch per foot (1.19°)
- 1/2 inch per foot (2.39°)
- Vertical: $\geq 45°$

Note that any slope less than 45 degrees is considered horizontal.

Horizontal Waste Pipes

When it comes to waste pipes, CMR 10.15 differentiates between a horizontal branch drain (handling some things) versus a horizontal building drain (handling everything). This leads to CMR 10.15 Table 2 as shown in Figure 1.10. Note the number of fixture units a pipe can handle has to do not only with the pipe's diameter, but also the *slope* of the pipe. (This wasn't a concern for pressurized water supply lines.)

Diameter of drain in inches	Horizontal Branch Drain (F.U.)	Building Drain		
		1/8 in./ft. (F.U.)	1/4 in./ft. (F.U.)	1/2 in./ft. (F.U.)
1½	3	---	---	---
2	6	---	---	---
2½	12	---	---	---
3	34*	---	40*	48*
4	160	180	216	250
5	360	390	480	575
6	620	700	840	1,000
8	1,400	1,600	1,920	2,300
10	2,500	2,900	3,500	4,200
12	3,900	4,600	5,600	6,700
15	7,000	8,300	10,000	12,000

* Not more than four water closets

Figure 1.10: CMR 10.15 Table 2: Maximum Loads in Fixture Units for Horizontal Drains.

Stacks (1 or 2 Intervals)

The story is different for vertical pipes. CMR 10.15 offers CMR 10.15 Table 3 as shown in Figure 1.11 for stacks having one or two branch intervals. The data in CMR 10.15 Table 3 seems to occur empiracally, which is to say the numbers were found by experiment rather than by formula. Thus the precise relationship between the stack diameter and the maximum load is not obvious without some work.

Diameter of Stack (inches)	Maximum Load on Stack (F.U.)
1½	4
2	8
2½	20
3*	48
4	240
5	540
6	930
8	2,100
10	3,750
12	5,850
15	10,500

Figure 1.11: CMR 10.15 Table 3: Maximum Loads in Fixture Units for Soil and Waste Stacks Having One or Two Branch Intervals.

Let y denote the maximum load on the stack in fixture units, and let x denote the pipe diameter in inches. Proposing

$$y = Ax^B \ ,$$

and running a piecewise power law analysis using the information from CMR 10.15 Table 3 leads to:

$$y = \begin{cases} x \leq 3 : & (0.81698)\, x^{3.5783} \\ x > 3 : & (5.5947)\, x^{2.8106} \end{cases}$$

Stacks (3+ Intervals)

For stacks having more than two branch intervals, we are provided with CMR 10.15 Table 4 as Figure 1.12.

Diameter of Stack	Number of Branch Intervals													Maximum Total Load for Stack
	3	4	5	6	7	8	9	10	11	12	13	14	15	
2	3	----	----	----	----	----	----	----	----	----	----	----	----	10
2 ½	8	7	----	----	----	----	----	----	----	----	----	----	----	28
3*	20	18	17	16	15	14	13	12	11	10	10	10	10	102
4	100	90	84	80	77	75	73	72	71	70	69	68	68	530
5	225	205	190	180	175	170	165	162	159	157	156	154	153	1,400
6	385	350	325	310	300	290	285	280	275	271	268	266	263	2,900
8	875	785	735	700	675	655	640	630	620	612	606	600	594	7,600
10	1,560	1,405	1,310	1,250	1,205	1,170	1,140	1,125	1,110	1,095	1,180	1,075	1,062	15,000
12	2,435	2,195	2,045	1,950	1,875	1,825	1,790	1,755	1,730	1,705	1,685	1,670	1,655	26,000
15	4,375	3,935	3,675	3,500	3,380	3,280	3,210	3,150	3,110	3,060	3,030	3,000	2,975	50,000

Figure 1.12: CMR 10.15 Table 4: Maximum Loads in Fixture Units for Any One Branch Interval on Multi-story Soil and Waste Stacks.

The lower body of CMR 10.15 Table 4, along with the data for stacks taller than 15 branch intervals, is generated from one formula. For a given stack diameter, we write

$$\text{Load (DFU)} = N\left(\frac{1}{2n} + \frac{1}{4}\right),$$

where N is the maximum load on the one- or two-interval stack reported in CMR 10.15 Table 3. The variable n is the number of branch intervals on the stack to be sized.

For example, for a stack size of 4-inch diameter with $n = 10$ branch intervals, use CMR 10.15 Table 3 to find $N = 240$ in drainage fixture units. The corresponding load is

$$L = 240\left(\frac{1}{2 \times 10} + \frac{1}{4}\right) = 72\,,$$

as properly found in CMR 10.15 Table 4.

For very tall buildings, the term $1/4$ dominates $1/2n$ and the above simplifies to

$$L \approx \frac{N}{4}\,,$$

which says the fixture unit load for any one branch in a very tall stack is about a quarter of the stack maximum in the one- or two-story case.

The right-most column in CMR 10.15 Table 4 is somewehat mysterious. To illustrate, suppose we have a building with 15 branch intervals using a stack diameter of 4 inches. According to the table, there are up to 68 fixture units allowed on any one branch. Planning optimistically, what if we want all 15 branches to handle 68 fixture units? Multiply the two to find $15 \times 68 = 1020$, which is about twice the maximum stack load according to the last column.

This descrepency lessens as we go down the table. Trying the 10 inch case, we see 1062×15 roughly matches the value in the last column. Beyond what's been demonstrated, however, there isn't an obvious pattern in the last column. Going for another power law analysis while and considering pipe diameters above three inches, the maximum total load as a function of diameter is approximately:

$$y = (5.7853)\, x^{3.3935}$$

2 Gas Pipe Sizing

2.1 Pipe Sizing Equations

In Revision 12/8/23 of CMR 10.00, the term 'gas' occurs 165 times. The term 'pipe' occurs 327 times. The combination 'gas pipe' occurs 0 times. Instead, one's first impression of gas pipe sizing must come from elsewhere, such as *NFPA ANSI Z223.1*, Chapter 6.

In this, the reader is alerted that *where the pipe size is to be determined using any of the three methods in 6.1.1 through 6.1.3, the diameter of each pipe segment shall be obtained from the pipe sizing tables in Section 6.2 or Section 6.3 or from the sizing equations in Section 6.4.*

The so-called three methods are the (i) longest length method, (ii) branch length method, (iii) and hybrid pressure method. Each heuristic varies slightly from the others and each may return different answer for the same input.

There do exist other methods of gas pipe sizing, including the less-often used 'pressure drop per 100 feet' method. Things also get wooly when adding new fixtures to existing systems. The going advice is when one calculation method gives an undesirable answer, try a different method.

Low Pressure Gas Formula

Appearing as item `6.4.1` is the *low pressure gas formula*, which describes gases below a pressure of 1.5 psi, or 10.3 kPa:

$$D = \frac{Q^{0.381}}{19.17\left(\Delta H/\left(Cr \cdot L\right)\right)^{0.206}} \tag{1.3}$$

In the above: (i) D is the inner diameter of the pipe measured in inches. (ii) Q is the volumetric flow rate through a pipe measured in cubic feet per hour in standard conditions. (iii) ΔH is a pressure drop measured in inches of water column. (This is a confusing unit that we'll have to take on below.) (iv) L is the length of pipe measured in feet.

The terms Cr, and as we'll also see, Y are uncolorfully called *formula factors*. Natural gas has $Cr = 0.6094$ and $Y = 0.9992$. Undiluted propane has $Cr = 1.2462$ and $Y = 0.9910$.

High Pressure Gas Formula

A variation on the low pressure gas formula is the *high pressure gas formula*, valid for pressures higher than pressure of 1.5 psi, or 10.3 kPa:

$$D = \frac{Q^{0.381}}{18.93\left(\left(P_1^2 - P_2^2\right)Y/\left(Cr \cdot L\right)\right)^{0.206}} \tag{1.4}$$

The terms P_1, P_2 are the respective upstream and downstream pressures, measured in psia.

One should feel suspicious when an equation contains any baked-in decimals, especially in exponents. It's irresistible, then, to figure out where the low- and high pressure gas equations come from.

2.2 Derivation of Formulas

The equations for gas pipe sizing originate from undergraduate-level thermodynamics. Particularly, we work within the *incompressible fluid* regime, in where the mass per unit volume is constant. Going for a minimalist derivation, we lay out only the important terms in what follows.

Hydraulic Diameter

Consider the motion of incompressible fluid through a straight pipe with cross sectional area A. For a notion of pipe diameter, define the *hydraulic diameter* D_H as

$$D_H = \frac{4A}{P} \,,$$

where P is the perimeter of the pipe. For the case of cylindrical pipes, we have

$$D_H = \frac{4\pi D^2}{4\pi D} = D \,,$$

which is just the circular diameter.

Volumetric Flow

Define the *wetted area* A_W as the portion of the cross-sectional area that is in contact with the moving fluid, i.e. the water flowing on the bottom of a pipe. The *volumetric flow*, denoted Q, is defined as

$$Q = A_W \left\langle v \right\rangle \,,$$

where $\langle v \rangle = v$ is the mean velocity of the fluid. Since gas occupies its entire container, we take A_W as the pipe's inner surface area, namely $\pi D L$, where L is the length.

Frictional Resistance

A model for the force of friction imposed on a moving fluid is attributed to Froude, providing the formula

$$F_R = f' \times A_W \times v^2 \,.$$

The coefficient f' is the frictional force per unit area with units to balance the v^2-term.

Bernoulli's Principle

A handy notion that we understand in terms of Newtonian laws and energy conservation is the famed *Bernoulli's principle*, stating that *for an incompressible fluid, the sum of its potential energy, pressure, and velocity remains constant.*

In the context of fluid moving through a pipe, one can expect that the effect of friction causes a loss of pressure ΔP between two given points in the pipe. Generally, one may write

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = \Delta P + P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \; ,$$

which accounts for the kinetic, gravitational potential, and pressure terms responsible for the motion of the fluid from point '1' to point '2'.

Head Loss

A quantity

$$H_f = \frac{\Delta P}{\rho g} \; ,$$

called the *head loss* is used to characterize the drop in pressure along a length of pipe. Dimensionally, we have

$$[H_f] = \frac{\text{kg m}}{\text{s}^2\,\text{m}^2}\frac{\text{m}^3\,\text{s}^2}{\text{kg m}} = \text{m} \; ,$$

i.e. pressure is interpreted as length (of water displaced in a column, for instance).

Darcy-Weisbach Equation

The effect of friction on a fluid moving in a pipe is characterized by the dimensionless *Darcy friction factor*, denoted f. To motivate this, consider the uniform motion of fluid at velocity v through a circular pipe of cross-sectional area A. With zero net acceleration, we use Newton's second law to establish

$$A\Delta P - F_R = 0 \; ,$$

or after simplifying,

$$H_f = \frac{f'}{\rho g}\frac{4}{D}Lv^2 \; .$$

The combination f'/ρ is dimensionless, which we take as proportional to the coefficient of friction for fluid moving through the pipe. To keep consistent with common literature on the topic, define

$$f = \frac{8f'}{\rho}$$

so the above is finally written

$$H_f = f\frac{L}{D}\frac{v^2}{2g} \; ,$$

known as the Darcy-Weisbach equation.

In terms of volumetric flow, the same equation is written

$$H_f = f\frac{8L}{\pi^2 g}\frac{Q^2}{D^5} \; .$$

If solving for the diameter of pipe involved, one has:

$$D = \left(\frac{f}{H_f}\frac{8L}{\pi^2 g}\right)^{1/5} Q^{2/5} \qquad (1.5)$$

From analysis alone, equation (1.5) is the closest we will get to the low- and high pressure gas Equations (1.3), (1.4).

Laminar Flow (Optional)

For the case of laminar flow, it can be shown that the friction factor f is exactly

$$f = \frac{64}{\text{Re}} \; ,$$

where Re is the *Reynold's number*, defined as

$$\text{Re} = \frac{\rho D}{\mu}\langle v\rangle \; ,$$

where μ is the viscosity of the fluid.

Engineering Fudge Factors

In terms of Q, H_f, L, the diameter of the pipe, according to Darcy-Weisbach via Euation (1.5), can be written

$$D = A \times Q^{0.4} H_f^{-0.2} L^{0.2} \; ,$$

where A is a factor not depending on Q, H_f, L.

Taking, for instance, the low pressure gas forumla and comparing coefficients, we see that the term 0.4 has been tweaked to 0.381, whereas 0.2 became 0.206, and so on. The high pressure gas formula, particularly the dependence on P^{-2} rather than P^{-1}, is the product of playing similar games with the equations of physics.

2.3 Fixture Demand

The reason we concern about gas pipe sizing is to supply enough fuel to each consuming fixture while not oversizing all supply lines to the point of hazard or waste.

A gas fixture's 'demand' is characterized by the average energy spent over time, measured in British thermal units per hour. Note that energy over time is formally equivalent to 'power'.

In the following we list several natural gas-consuming fixtures with the respective energy consumption rate. Numeric values are hashed together from a variety of teaching and real-world sources. All are subject to change. (The term 'domestic' is abbreviated as 'dom.')

- Dryer (clothing, dom.) (30-35 kBtu/hr)
- Fireplace (direct vent.) (30-40 kBtu/hr)
- Furnace (100 kBtu/hr)
- Hydronic boiler (dom.) (100-120 kBtu/hr)
- Pool Heater (4 gal./min.) (285 kBtu/hr)
- Stove (freestanding range, dom.) (65 kBtu/hr)
- Water Heater (40 gal. dom.) (35-40 kBtu/hr)
- Water Heater (50 gal. dom.) (50 kBtu/hr)

Calorific Value

Gas-consuming fixtures are usually characterized by energy consumption rate in British thermal units per hour. However this information is often reported as the volume of gas consumed in cubic feet per hour (`cfh`). To convert between the amount of energy (Btu) in a gas and the cubic-foot volume (ft^3) of the gas, one needs to know the *calorific value* of the gas.

By a conspiracy between engineers and God, it just happens that the calorific value of natural gas is about one thousand British thermal units per cubic foot:

$$\text{C.V. of natural gas} \approx 1000\,\mathrm{Btu/ft^3}$$

For this reason, we can usually fudge the unit kBtu/hr as being effectively equal to ft^3/hr. Of course, the calorific value depends on a few variables that, for our purposes, are held standard and are baked into the number 1000.

Such a shortcut doesn't work for other gases such as propane, or even natural gas under different conditions. Some texts and manuals use the calorific value of 1100 Btu/ft^3. Using this, we may convert some number X Btu/hr into Y cubic feet per hour by writing:

$$Y = \frac{X\,\cancel{\mathrm{Btu}}/\mathrm{hr}}{1100\,\cancel{\mathrm{Btu}}/\mathrm{ft}^3} = \frac{X}{1100}\frac{\mathrm{ft}^3}{\mathrm{hr}}$$

2.4 Schematics

Any gas distribution system can be represented in a diagram, usually two-dimensional, much like a schematic for an electric circuit. The so-called pipe sizing schematic contains at least one point of delivery for gas service, along with all gas-consuming fixtures represented as locations.

Connecting the point(s) of service to all fixtures is a network of pipes represented as lines as shown in Figure 1.13.

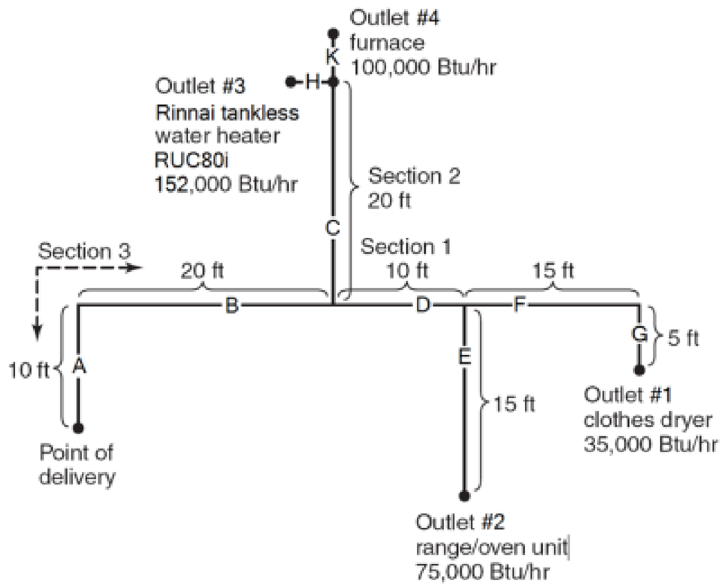


Figure 1.13: Sample gas distribution schematic. (*Rinnai Technical Bulletin 111-1/2". Gas Line Piping Information.* [a])

[a]All Information was obtained from National Fuel Gas Code, NFPA54, ANSI Z223.1.

It's not clear whether the Rinnai company originated the diagram published. The *2010 Oregon Mechanical Specialty Code*, in Appendix C-A, offers a suspisciously similar schematic shown in Figure 1.14.

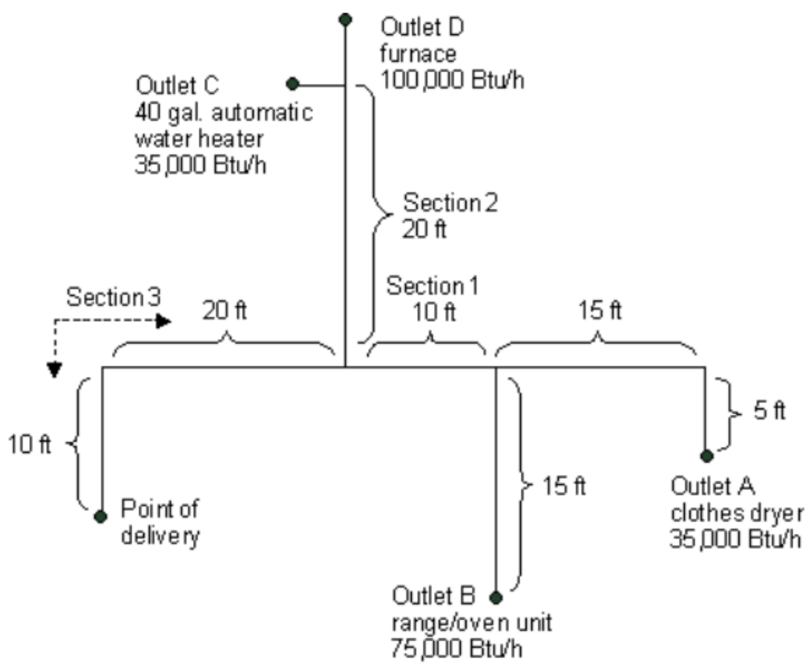


Figure 1.14: Sample gas distribution schematic. (*2010 Oregon Mechanical Specialty Code. Figure C-A.7.1. Piping Plan Showing a Steel Piping System*)

A very similar diagram appears in the literature of El Dorado County, California as shown in Figure 1.15. The engineers responsible for these diagrams must have cheated off the same guy.

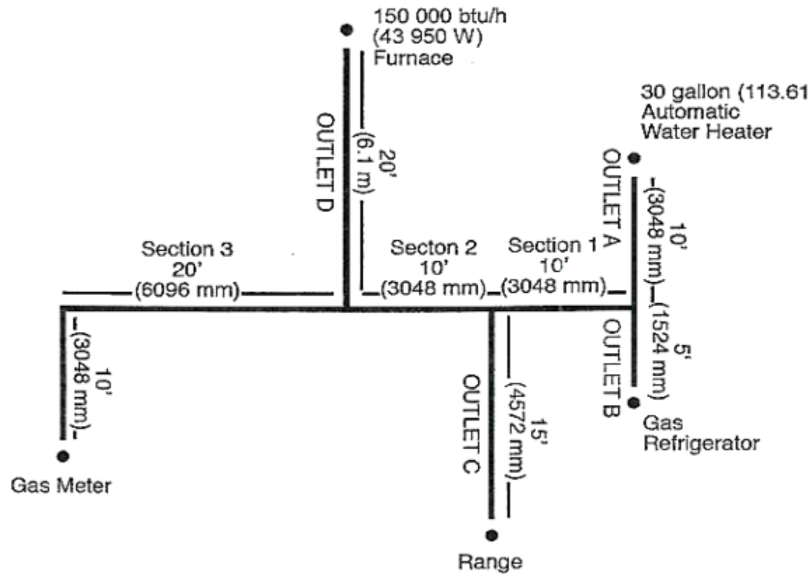


Figure 1.15: Sample gas distribution schematic. (*El Dorado County, CA. GAS PIPE LINE CALCULATION SIZING. Figure A*)

Some new symbols for the gas meter, distribution manifold, shut-off valve, and pressure regulator are shown in Figures 1.16-1.17. In each Figure, the fixture demands are listed in cubic feet per hour rather than British thermal units per hour. (Beware this difference!)

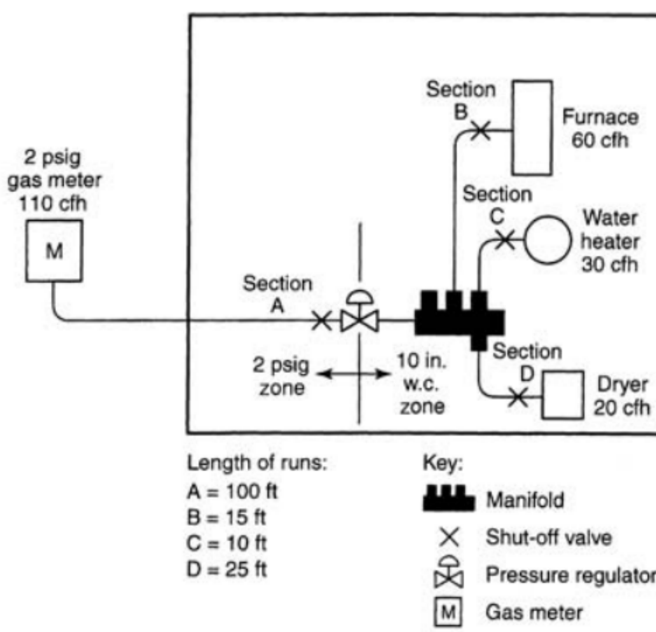


Figure 1.16: Sample gas distribution schematic. (*2010 Oregon Mechanical Specialty Code. Figure C-A.7.2. Piping Plan Showing a CSST System*)

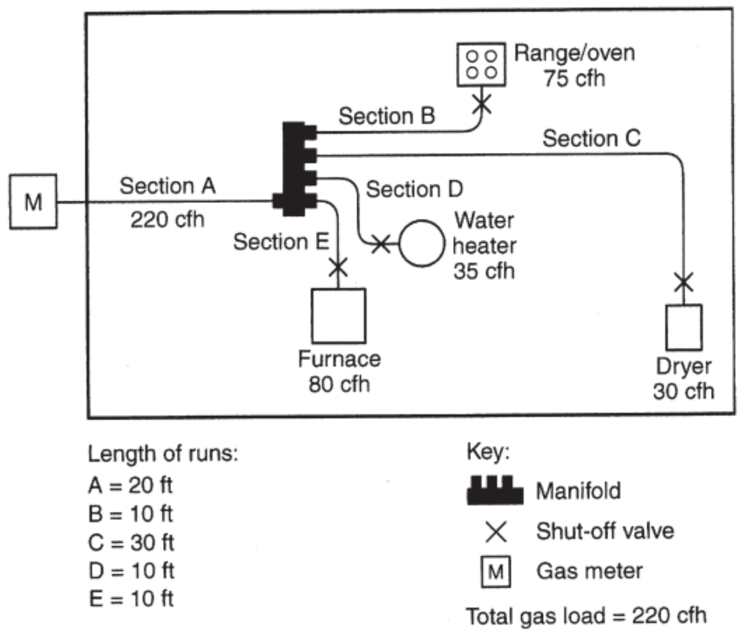


Figure 1.17: Sample gas distribution schematic. (*2010 Oregon Mechanical Specialty Code. Figure C-A.7.3. Piping Plan Showing a Copper Tubing System*)

2.5 Preparing a Schematic

When confronted with a gas pipe sizing problem, the diagram is everything. However, these may be handed to you with too much or too little information on the page. To make a clean preparation, let us do some edits on Figure 1.14 (example alpha) and Figure 1.15 (example beta).

For starters, ignore any extraneous information such as 'section' number. The point of delivery is denoted `M` (for meter) with an arrow indicating direction of flow as shown in Figures 1.18, 1.19.

Label all Fixtures (A, B, C, ...)

If not present in the schematic, write the resource demand in ft^3/hr or kBtu/hr of each fixture. Each fixture, or 'outlet', should take a unique label such as A, B, C, etc.

Label all Tees (X, Y, Z, ...)

It also helps to label each tee connection in the system with a different grouping of letters than those reseved for fixtures. For these, it suffices to use the labels X, Y, Z, etc.

Label the Meter (M)

Apart from 'M' standing for 'meter', it's convenient that 'M' stands in the middle of the alphabet, dividing fixtures (A, B, ...) from tees (X, Y, ...).

Turns and Couplings

For our purposes, there is no need to label any ninety-degree connections or similar. The pipe diameter is assumed constant across such fittings.

However, when there are a large number of accessories involved, i.e. elbows, valves, etc., there is reason to account for the friction introduced at each. In a sentence, the effect of a given accessory is characterized by the some length of pipe that would impose the same effect as the accessory.

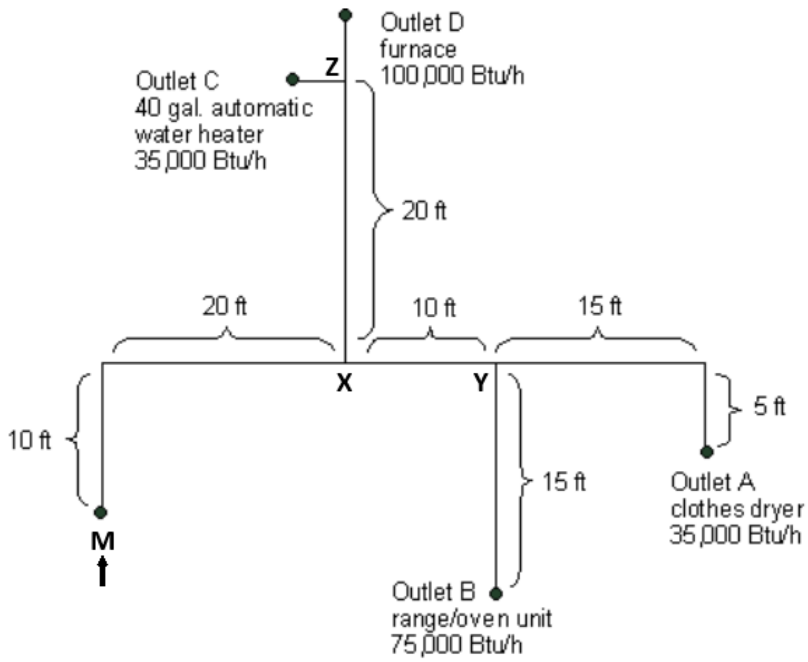


Figure 1.18: Example alpha: Sample schematic (Figure 1.14) prepared for pipe sizing.

Worksheet alpha				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	60	35 from Y using ?	-	-
B	55	75 from Y using ?	-	-
C	50	35 from Z using ?	-	-
D	50	100 from Z using ?	-	-
M	0	-	245 to X	-
X	30	245 from M using ?	110 to Y	135 to Z
Y	40	110 from X using ?	35 to A	75 to B
Z	50	135 from X using ?	35 to C	100 to D

Table 1.1: Example alpha worksheet: Representation of Figure 1.18.

Prepare Worksheet

Before choosing any sizing method, it's helpful to determine all pertinent lengths and flow rates throughout the system. This can be done on the schematic itself, however we'll develop the habit of capturing schematics like Figure 1.18 in worksheet form as shown. (Note fixtures C and D are each directly connected to Z. That is, each is located 20 ft from X.)

Reading the worksheet, one observes:

- The worksheet encodes the essential information on the schamatic.
- The objective is to turn question marks (?) into pipe diameters in inches.
- The 'Input', 'Out 1', and 'Out 2' columns are reported in ft^3/hr, i.e. cfh.
- The sum of all fixture inputs equals the meter output.
- For a given tee connection, the input equals the sum of the outputs.
- The words 'to' and 'from' occur in equal numbers.
- The input to the meter and the output from each fixture is ignored.

Performing a similar task on the second example (beta) gives rise to Figure 1.19 and the corresponding worksheet:

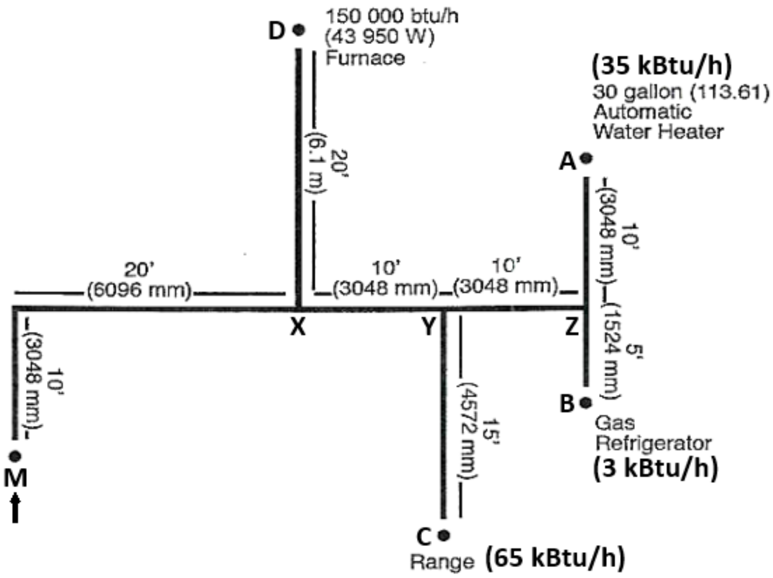


Figure 1.19: Example beta: Sample schematic (Figure 1.15) prepared for pipe sizing.

Worksheet beta				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	60	35 from Z using ?	-	-
B	55	3 from Z using ?	-	-
C	55	65 from Y using ?	-	-
D	50	150 from X using ?	-	-
M	0	-	253 to X	-
X	30	253 from M using ?	150 to D	103 to Y
Y	40	103 from X using ?	65 to C	38 to Z
Z	50	38 from Y using ?	35 to A	3 to B

Table 1.2: Example beta worksheet: Representation of Figure 1.19.

To prepare another schematic (example gamma), we turn to the *Plumbing-Heating-Cooling Contractors of Massachusetts*, or *PHCC*, for a study guide called *Introduction: PSI Exams.* Figure 1.20 contains a gas pipe sizing problem adapted from Slide 291. All fixtures, tees, and pipe lengths are labeled.

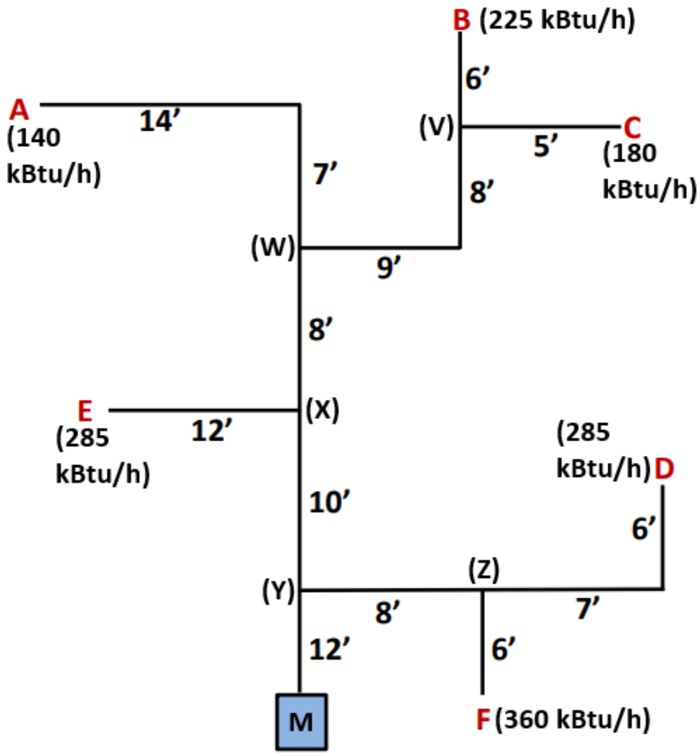


Figure 1.20: Example gamma: Sample schematic perpared for pipe sizing. Adapted from *PHCC. Introduction: PSI Exams. Slide 291.*

Worksheet gamma				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	51	140 from W using ?	-	-
B	53	225 from V using ?	-	-
C	52	180 from V using ?	-	-
D	33	285 from Z using ?	-	-
E	34	285 from X using ?	-	-
F	26	360 from Z using ?	-	-
M	0	-	1475 to Y	-
V	47	405 from W using ?	225 to B	180 to C
W	30	545 from X using ?	140 to A	405 to V
X	22	830 from Y using ?	285 to E	545 to W
Y	12	1475 from M using ?	830 to X	645 to Z
Z	20	645 from Y using ?	285 to D	360 to F

Table 1.3: Example gamma worksheet: Representation of Figure 1.20.

In each example above (alpha, beta, gamma), assume all pipes carry natural gas under conditions consistent with *ANSI Z223.1*, Table 6.2(b). Schedule 40 Metallic Pipe. (See Figure 1.21.)

2.6 Pipe Sizing Charts

Throughout *ANSI Z223.1* are various pipe sizing charts for natural gas in various conditions. The reason for having charts is to save the plumber from directly applying Equations (1.3) and (1.4) in the field.

Header

Shown in Figures 1.21, 1.22 are samples from *ANSI Z223.1*, indicitave of all Tables. Every bit of information on a Table is relevant, especially the header.

Before considering the body of the chart, the header contains six bits of information (reading from Table 6.2(b) in Figure 1.21):

- Title, Table 6.2(b).
- Material to be used: sch. 40 metallic
- Type of gas: natural
- Inlet pressure: less than 2 psi
- Pressure drop: 0.5 in. w. c. (This is the head loss H_f.)
- Specific gravity: 0.60 (Specific gravity is the ratio of the density of gas to the density of atmosphere.)

Note the inlet pressure is below 2 psi, thus we assume the low pressure gas formula will be behind the numbers.

GAS:	**NATURAL**												
INLET PRESSURE:	**LESS THAN 2 PSI**												
PRESSURE DROP:	**0.5 in w.c.**												
SPECIFIC GRAVITY:	**0.60**												
	PIPE SIZE (inch)												
NOMINAL:	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	4	5	6	8	10	12
ACTUAL ID:	0.622	0.824	1.049	1.380	1.610	2.067	2.469	3.068	4.026	5.047	6.065	7.981	10.020	11.938
LENGTH (ft)	**CAPACITY IN CUBIC FEET OF GAS PER HOUR**													
10	172	360	678	1,390	2,090	4,020	6,400	11,300	23,100	41,800	67,600	139,000	252,000	399,000
20	118	247	466	957	1,430	2,760	4,400	7,780	15,900	28,700	46,500	95,500	173,000	275,000
30	95	199	374	768	1,150	2,220	3,530	6,250	12,700	23,000	37,300	76,700	139,000	220,000
40	81	170	320	657	985	1,900	3,020	5,350	10,900	19,700	31,900	65,600	119,000	189,000
50	72	151	284	583	873	1,680	2,680	4,740	9,660	17,500	28,300	58,200	106,000	167,000
60	65	137	257	528	791	1,520	2,430	4,290	8,760	15,800	25,600	52,700	95,700	152,000
70	60	126	237	486	728	1,400	2,230	3,950	8,050	14,600	23,600	48,500	88,100	139,000
80	56	117	220	452	677	1,300	2,080	3,670	7,490	13,600	22,000	45,100	81,900	130,000
90	52	110	207	424	635	1,220	1,950	3,450	7,030	12,700	20,600	42,300	76,900	122,000
100	50	104	195	400	600	1,160	1,840	3,260	6,640	12,000	19,500	40,000	72,600	115,000

Figure 1.21: Sample from *ANSI Z223.1*, Chapter 6. Table 6.2(b). Schedule 40 Metallic Pipe.

Axes

The axes of all Tables have pipe length charted vertically and pipe diameter charted horizontally. As for pipe diameter, we tend to ignore the 'Actual ID' row, reading instead from the 'nominal' row.

Body

Any cell in the body of the Table is some number measured in ft^3/hr, which, for natural gas, is sometimes interchangeable with kBtu/hr. For a given demand, read to the left axis for the pipe length, and read to the top axis for the pipe diameter.

Table 6.2.1(j) Semirigid Copper Tubing

	Gas:	Natural
	Inlet Pressure:	Less than 2 psi
	Pressure Drop:	1.0 in. w.c.
	Specific Gravity:	0.60

INTENDED USE: Tube Sizing Between House Line Regulator and the Appliance.

		Tube Size (in.)								
Nominal:	K & L:	1/4	3/8	1/2	5/8	3/4	1	1 1/4	1 1/2	2
	ACR:	3/8	1/2	5/8	3/4	7/8	1 1/8	1 3/8	—	—
Outside:		0.375	0.500	0.625	0.750	0.875	1.125	1.375	1.625	2.125
Inside:*		0.305	0.402	0.527	0.652	0.745	0.995	1.245	1.481	1.959
Length (ft)		Capacity in Cubic Feet of Gas per Hour								
10		39	80	162	283	402	859	1,550	2,440	5,080
20		27	55	111	195	276	590	1,060	1,680	3,490
30		21	44	89	156	222	474	853	1,350	2,800
40		18	38	77	134	190	406	730	1,150	2,400
50		16	33	68	119	168	359	647	1,020	2,130

Figure 1.22: Sample from *ANSI Z223.1*, Chapter 6. Table 6.2.1(j). Semirigid Copper Tubing.

2.7 Longest Length Method

Quoting directly from *ANSI Z223.1*, Chapter 6, the longest length method is stated as:

The pipe size of each section of gas piping shall be determined using the longest length of piping from the point of delivery to the most remote outlet and the load of the section.

Implementing the longest length method goes as follows:

1. Determine which fixture is furthest distance from the meter in developed length. This is the so-called longest length. (This is also the last time you count distances to fixtures in this method.)

2. Determine the total ft^3/hr demand of each fixture and of the whole system. (Convert from kBtu/hr if necessary.)

3. In the appropriate Table such as 6.2(b), read down the left column until finding the longest length from Step 1. Round your search to the next row if necessary, but never round down in pipe size.

4. Staying in the longest length row, read across the Table until finding the total ft^3/hr demand from Step 2. Round your total up to the next column if necessary.

5. At this step you're pointing at one cell in the body of the Table. The header of the column indicates the diameter of the gas pipe leaving the meter.

6. All other pipe sizes are derived from the *same* row in the Table. To size any given pipe segment, determine the total demand at the end of that segment and read the Table as done in Steps 3-5.

7. Tip: Deal with all fixtures first. Then treat tee fittings as if they're fixtures.

Worked Example: Alpha

Apply the longest length method to example alpha represented in Figure 1.18. The two pipes leaving Z are assumed to have negligible length.

Referencing the Figure (namely the worksheet beneath it), the longest length corresponds to outlet A at 60 ft from meter. The total demand of the system is 245 ft^3/hr. (Steps 1 and 2.)

Looking in the Table 6.2(b) for a length of 60 ft and a demand of 245 ft^3/hr leads to the cell labeled 257ft^3/hr. The header of this column indicates 1 inch. The pipe diameter leaving the meter is 1 inch. (Steps 3, 4, and 5.)

In ft^3/hr, the fixtures A, B, C, D receive 35, 75, 35, 100 respectively. Staying in the 60 ft row of the Table, we straightforwardly read off the diameters going to each fixture. In inches, the results are 1/2, 3/4, 1/2, 3/4.

The tee labeled X receives 245 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 245 ft^3/hr leads to the cell labeled 257 ft^3/hr. The header of this column indicates 1 inch. The pipe diameter entering X from M is 1 inch.

The tee labeled Y receives 110 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 110 ft^3/hr leads to the cell labeled 137 ft^3/hr. The header of this column indicates 3/4 inch. The pipe diameter entering Y from X is 3/4 inch.

The tee labeled Z receives 135 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 135 ft^3/hr leads to the cell labeled 137 ft^3/hr. The header of this column indicates 3/4 inch. The pipe diameter entering Z from X is 3/4 inch.

All pipes are now sized and the problem is solved. The updated worksheet is as follows:

Worksheet alpha: Longest length method (using Table 6.2(b))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	60	35 from Y using 1/2 in	-	-
B	55	75 from Y using 3/4 in	-	-
C	50	35 from Z using 1/2 in	-	-
D	50	100 from Z using 3/4 in	-	-
M	0	-	245 to X	-
X	30	245 from M using 1 in	110 to Y	135 to Z
Y	40	110 from X using 3/4 in	35 to A	75 to B
Z	50	135 from X using 3/4 in	35 to C	100 to D

Worked Example: Beta

Apply the longest length method to example beta represented in Figure 1.19.

Referencing the Figure (namely the worksheet beneath it), the longest length corresponds to outlet A at 60 ft from meter. The total demand of the system is 253 ft^3/hr. (Steps 1 and 2.)

Looking in the Table 6.2(b) for a length of 60 ft and a demand of 253 ft^3/hr leads to the cell labeled 257ft^3/hr. The header of this column indicates 1inch. (Steps 3, 4, and 5.) The pipe diameter leaving the meter is 1 inch.

In ft^3/hr, the fixtures A, B, C, D receive 35, 3, 65, 150 respectively. Staying in the 60ft row of the Table, we straightforwardly read off the diameters going to each fixture. In inches, the results are 1/2, 1/2, 1/2, 1.

The tee labeled X receives 253 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 253 ft^3/hr leads to the cell labeled 257 ft^3/hr. The header of this column indicates 1 inch. The pipe diameter entering X from M is 1 inch.

The tee labeled Y receives 103 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 103 ft^3/hr leads to the cell labeled 137 ft^3/hr. The header of this column indicates 3/4 inch. The pipe diameter entering Y from X is 3/4 inch.

The tee labeled Z receives 38 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 38 ft^3/hr leads to the cell labeled 65 ft^3/hr. The header of this column indicates 1/2 inch. The pipe diameter entering Z from Y is 1/2 inch.

From this point we get to cheat the rest. All pipe diameters downstream of Y are 1/2 inch.

Worksheet beta: Longest length method (using Table 6.2(b))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	60	35 from Z using 1/2 in	-	-
B	55	3 from Z using 1/2 in	-	-
C	55	65 from Y using 1/2 in	-	-
D	50	150 from X using 1 in	-	-
M	0	-	253 to X	-
X	30	253 from M using 1 in	150 to D	103 to Y
Y	40	103 from X using 3/4 in	65 to C	38 to Z
Z	50	38 from Y using 1/2 in	35 to A	3 to B

Worked Example: Gamma

Apply the longest length method to example gamma represented in Figure 1.20.

Referencing the Figure (namely the worksheet beneath it), the longest length corresponds to outlet B at 53 ft from meter. The total demand of the system is 1475 ft^3/hr. (Steps 1 and 2.)

Looking in the Table 6.2(b) for a length of 60 ft and a demand of 1475 ft^3/hr leads to the cell labeled 1520 ft^3/hr. The header of this column indicates 2 inch. (Steps 3, 4, and 5.) The pipe diameter leaving the meter is 2 inch. Therefore segment MY is 2 inch.

In ft^3/hr, the fixtures A, B, C, D, E, F receive 140, 225, 180, 285, 285, 360, respectively. Staying in the 60 ft row of the Table, we straightforwardly read off the diameters going to each fixture. In inches, the results are 1, 1, 1, 1.25, 1.25, 1.25.

The tee labeled V receives 405 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 405 ft^3/hr leads to the cell labeled 528 ft^3/hr. The header of this column indicates 1.25 inch. The pipe diameter entering V from W is 1.25 inch.

The tee labeled W receives 545 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 545 ft^3/hr leads to the cell labeled 791 ft^3/hr. The header of this column indicates 1.5 inch. The pipe diameter entering W from X is 1.5 inch.

The tee labeled X receives 830 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 830 ft^3/hr leads to the cell labeled 1520 ft^3/hr. The header of this column indicates 2 inch. The pipe diameter entering X from Y is 2 inch.

The tee labeled Z receives 645 ft^3/hr. Looking in the Table for a length of 60 ft and a demand of 645 ft^3/hr leads to the cell labeled 791 ft^3/hr. The header of this column indicates 1.5 inch. The pipe diameter entering Z from Y is 1.5 inch.

All pipes are now sized and the problem is solved. The updated worksheet is as follows:

Worksheet gamma: Longest length method (using Table 6.2(b))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	51	140 from W using 1 in	-	-
B	53	225 from V using 1 in	-	-
C	52	180 from V using 1 in	-	-
D	33	285 from Z using 1.25 in	-	-
E	34	285 from X using 1.25 in	-	-
F	26	360 from Z using 1.25 in	-	-
M	0	-	1475 to Y	-
V	47	405 from W using 1.25 in	225 to B	180 to C
W	30	545 from X using 1.5 in	140 to A	405 to V
X	22	830 from Y using 2 in	285 to E	545 to W
Y	12	1475 from M using 2 in	830 to X	645 to Z
Z	20	645 from Y using 1.5 in	285 to D	360 to F

Problems

Problem 1

Let A, B, C, D represent gas-consuming fixtures that operate under conditions consistent with Table 6.2(b). In particular:

- A = Furnace (100 kBtu/hr)
- B = Water heater (40 kBtu/hr)
- C = Dryer (30 kBtu/hr)
- D = Fireplace (30 kBtu/hr)

Assume the calorific value of natural gas is 1000 Btu/ft^3. If the system is arranged according to the following worksheet, calculate all pipe diameters using the longest length method.

Longest length method (using Table 6.2(b))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	22	100 from X using ?	-	-
B	21	40 from Y using ?	-	-
C	28	30 from Z using ?	-	-
D	62	30 from Z using ?	-	-
M	0	-	200 to X	-
X	14	200 from M using ?	100 to A	100 to Y
Y	16	100 from X using ?	40 to B	60 to Z
Z	22	60 from Y using ?	30 to C	30 to D

Answer: Longest length 62 ft, round up to 70 ft.

- A: 100 from X using 3/4 in
- X: 200 from M using 1 in
- Y: 100 from X using 3/4 in
- All other diameters 1/2 in.

Problem 2

For systems that tolerate a pressure drop of 0.3 in. w.c. instead of 0.5, the Table we've been using must be replaced with Table 6.2(a), shown in Figure 1.23.

Table 2: **(Schedule 40 Metallic Pipe)**					***Gas:***		***Natural***	
					Inlet Pressure:		***Less than 2 psi***	
					Pressure Drop:		***0.3 in. w.c.***	
					Specific Gravity:		***0.6***	
N/A: A flow of less than 10 cfh; All table entres round to 3 significant digits; BTUH = CFH X 1000								
	Pipe Size (in)							
Moninal:	*1/2*	*3/4*	*1*	*1 1/4*	*1 1/2*	*2*	*2 1/2*	*3*
Actual ID:	*0.622*	*0.824*	*1.049*	*1.380*	*1.610*	*2.067*	*2.469*	*3.068*
Length (ft)	***Capacity in Cubic Feet of Gas per Hour (CFH)***							
10	131	273	514	1060	1580	3050	4860	8580
20	90	188	353	726	1090	2090	3340	5900
30	72	151	284	583	873	1680	2680	4740
40	62	129	243	499	747	1440	2290	4050
50	55	114	215	442	662	1280	2030	3590
60	50	104	195	400	600	1160	1840	3260
70	46	95	179	368	552	1060	1690	3000
80	42	89	167	343	514	989	1580	2790
90	40	83	157	322	482	928	1480	2610
100	38	79	148	304	455	877	1400	2470

Figure 1.23: Sample from *ANSI Z223.1*, Chapter 6. Table 6.2(a). Schedule 40 Metallic Pipe.

Let A, B, C, D, E represent gas-consuming fixtures that operate under conditions consistent with Table 6.2(b). In particular:

- A = Furnace (100 kBtu/hr)
- B = Water heater (40 kBtu/hr)
- C = Dryer (30 kBtu/hr)
- D = Fireplace (30 kBtu/hr)
- E = Stove (65 kBtu/hr)

Assume the calorific value of natural gas is 1000 Btu/ft^3. If the system is arranged according to the following worksheet, calculate all pipe diameters using the longest length method.

Longest length method (using Table 6.2(a))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	32	100 from X using ?	-	-
B	30	40 from Y using ?	-	-
C	28	30 from Z using ?	-	-
D	40	30 from Z using ?	-	-
E	37	65 from Z using ?	-	-
M	0	-	265 to X	-
W	26	140 from X using ?	100 to A	40 to B
X	16	265 from M using ?	140 to W	125 to Y
Y	23	125 from X using ?	30 to C	95 to Z
Z	28	95 from Y using ?	30 to D	65 to E

Answer: Longest length 40 ft.

- A: 100 from X using 3/4 in
- B: 40 from Y using 1/2 in
- C: 30 from Z using 1/2 in
- D: 30 from Z using 1/2 in
- E: 65 from Z using 3/4 in
- W: 140 from X using 1 in
- X: 265 from M using 1.25 in
- Y: 125 from X using 3/4 in
- Z: 95 from Y using 3/4 in

2.8 Branch Length Method

Quoting directly from *ANSI Z223.1*, Chapter 6, the branch length method is stated as:

---

1. Pipe size of each section of the longest pipe run from the point of delivery to the most remote outlet shall be determined using the longest run of piping and the load of the section.
2. The pipe size of each section of branch piping not previously sized shall be determined using the length of piping from the point of delivery to the most remote outlet in each branch and the load of the section.

Implementing the branch length method goes as follows:

1. Choose a fixture and determine its distance from the meter in developed length. (This number changes per fixture.)
2. Determine that fixture's demand in ft^3/hr. (Convert from kBtu/hr if necessary.)
3. In the appropriate Table such as 6.2(b), read down the left column until finding the length from Step 1. Round your search to the next row if necessary, but never round down in pipe size.
4. Staying in the length row, read across the Table until finding the total ft^3/hr demand from Step 2. Round your total up to the next column if necessary.
5. At this step you're pointing at one cell in the body of the Table. The header of the column indicates the diameter of the gas pipe serving the fixture.
6. Repeat for all fixtures.
7. Treat tee fittings as if they're fixtures with the following extra rule: the effective distance from the meter to a tee fitting includes the length of pipe from the fitting to the most remote outlet after the fitting.

The branch length method involves more fiddling with pipe lengths in the schematic. This process can get wooly, thus it's strongly recommended to generate a worksheet or mark the schematic with all pertinent information. In particular (reiterating Step 7), **the effective distance from any tee fitting to the meter must include the distance to the most remote outlet downstream of the fitting**.

Reworked Examples

Let us rework Examples alpha, beta, gamma using the branch length method. Starting with the same worksheets under Figures 1.18, 1.19, 1.20, use Steps 1-7 to reason:

Worksheet alpha: Branch length method (using Table 6.2(b))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	60	35 from Y using 1/2 in	-	-
B	55	75 from Y using 3/4 in	-	-
C	50	35 from Z using 1/2 in	-	-
D	50	100 from Z using 3/4 in	-	-
M	0	-	245 to X	-
X	30 (+30)	245 from M using 1 in	110 to Y	135 to Z
Y	40 (+20)	110 from X using 3/4 in	35 to A	75 to B
Z	50 (+0)	135 from X using 3/4 in	35 to C	100 to D

Worksheet beta: Branch length method (using Table 6.2(b))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	60	35 from Z using 1/2 in	-	-
B	55	3 from Z using 1/2 in	-	-
C	55	65 from Y using 1/2 in	-	-
D	50	150 from X using 3/4 in	-	-
M	0	-	253 to X	-
X	30 (+30)	253 from M using 1 in	150 to D	103 to Y
Y	40 (+20)	103 from X using 3/4 in	65 to C	38 to Z
Z	50 (+10)	38 from Y using 1/2 in	35 to A	3 to B

Worksheet gamma: Branch length method (using Table 6.2(b))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	51	140 from W using 1 in	-	-
B	53	225 from V using 1 in	-	-
C	52	180 from V using 1 in	-	-
D	33	285 from Z using 1 in	-	-
E	34	285 from X using 1 in	-	-
F	26	360 from Z using 1 in	-	-
M	0	-	1475 to Y	-
V	47 (+6)	405 from W using 1.25 in	225 to B	180 to C
W	30 (+23)	545 from X using 1.5 in	140 to A	405 to V
X	22 (+31)	830 from Y using 2 in	285 to E	545 to W
Y	12 (+41)	1475 from M using 2 in	830 to X	645 to Z
Z	20 (+13)	645 from Y using 1.25 in	285 to D	360 to F

In Example alpha, the results attained by the branch length method are the same as those attained with the longest length method.

In Example beta, all results agree with the longest length method with one exception, and that is the pipe size leading to outlet D. To double check, we have that outlet D is 50 ft from the meter while demanding 150 ft^3/hr, which, according to the branch length method, requires 3/4 inch pipe as calculated. This number came out to 1 inch using the longest length method, namely because we were trapped in the 60 ft row.

Problems

Problem 3

Let A, B, C, D, E represent gas-consuming fixtures that operate under conditions consistent with Table 6.2(b). In particular:

- A = Pool heater (250 kBtu/hr)
- B = Boiler (100 kBtu/hr)
- C = Fireplace (30 kBtu/hr)
- D = Stove (365 kBtu/hr)
- E = Water heater (40 kBtu/hr)

Assume the calorific value of natural gas is 1000 Btu/ft^3. If the system is arranged according to the following worksheet, calculate all pipe diameters using the branch length method.

Branch length method (using Table 6.2(b))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	163	250 from W using ?	-	-
B	28	100 from X using ?	-	-
C	38	30 from Y using ?	-	-
D	44	65 from Z using ?	-	-
E	54	40 from Z using ?	-	-
M	0	-	485 to W	-
W	13 (+150)	485 from M using ?	250 to A	235 to X
X	18 (+37)	235 from W using ?	100 to B	135 to Y
Y	28 (+27)	135 from X using ?	30 to C	105 to Z
Z	34 (+20)	105 from Y using ?	65 to D	40 to E

Answer:

- A: 250 from W using 1.25 inch
- B: 100 from X using 3/4 inch
- C: 30 from Y using 1/2 inch
- D: 65 from Z using 1/2 inch
- E: 40 from Z using 1/2 inch
- W: 485 from M using 2 inch
- X: 235 from W using 1 inch
- Y: 135 from X using 3/4 inch
- Z: 105 from Y using 3/4 inch

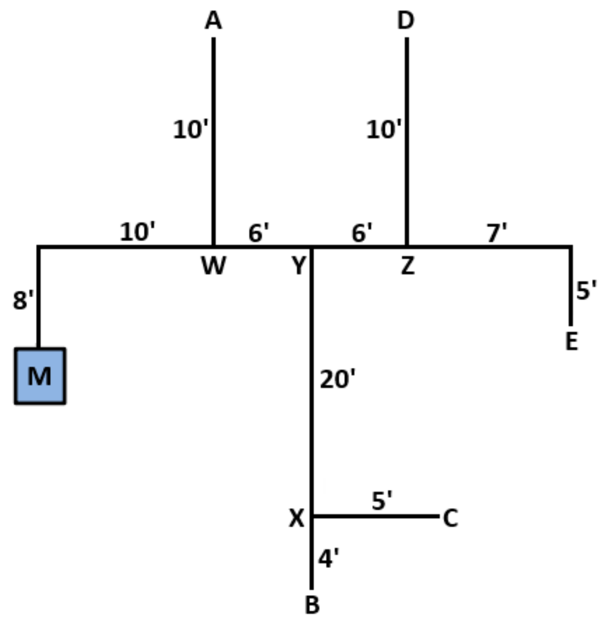
Problem 4

Let A, B, C, D, E represent gas-consuming fixtures that operate under conditions consistent with Table 6.2(b). In particular:

- A = Stove (65 kBtu/hr)
- B = Furnace (100 kBtu/hr)
- C = Water heater (40 kBtu/hr)
- D = Dryer (30 kBtu/hr)
- E = Fireplace (30 kBtu/hr)

Assume the calorific value of natural gas is 1000 Btu/ft^3.

The system is depicted in the accompanying Figure. Distances not to scale. For the longest length method and the branch length method, generate worksheets representing the schematic and size the system.



Longest length or Branch length method (using Table 6.2(b))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
A	28	65 from W using 1/2 in	-	-
B	48	100 from X using 3/4 in	-	-
C	49	40 from X using 1/2 in	-	-
D	40	30 from Z using 1/2 in	-	-
E	42	30 from Z using 1/2 in	-	-
M	0	-	265 to W	-
W	18 (+31)	265 from M using 1 in	65 to A	200 to Y
X	44 (+5)	140 from Y using 3/4 in	100 to B	40 to C
Y	24 (+25)	200 from W using 1 in	140 to X	60 to Z
Z	30 (+12)	60 from Y using 1/2 in	30 to D	30 to E

Problem 5

Consider the system depicted in Figure 1.17. Ignoring the effect of any shut-off valves, the central player in the schematic is a manifold, denoted X, which is essentially a tee with many branches.

The system is to be piped with semirigid copper tubing sustaining a pressure drop at 1 in. w. c. The gas has a specific gravity of 0.60 and the calorific valus of the gas is 1000 Btu/hr.

Given the material and conditions, it is warranted to size the system using Table 6.2.1(j) as depicted in Figure 1.22. Create an appropriate worksheet and size the system using the branch length method.

Answer: For the effective distance to X, use 20 ft plus the distance to the furthest outlet, i.e. 30 ft.

Branch length method (using Table 6.2.1(j))				
Symbol	**From Meter** (ft)	**Input** (cfh)	**Out 1** (cfh)	**Out 2** (cfh)
B	30	75 from X using 1/2 in	-	-
C	50	30 from X using 3/8 in	-	-
D	30	35 from X using 3/8 in	-	-
E	30	80 from X using 1/2 in	-	-
M	0	-	220 to X	-
X	20 (~~+10~~)	220 from M using 1 in	75 to B	-
X	20 (+30)	-	30 to C	-
X	20 (~~+10~~)	-	35 to D	-
X	20 (~~+10~~)	-	80 to E	-

2.9 Other Sizing Methods

There are several ways to solve the gas pipe sizing problem, but arguably the longest length method and branch lengthod are the most practical in the field.

When there is a pressure regular in the system, as sketched in Figure 1.16, then the **Hybrid pressure method** must be used. In such a case, the system is divided into (at least) one high-pressure section and one-low pressure section, each following its own sizing procedure.

There also exists the **Pressure drop per 100 feet method**. This method considers the loss in pressure due to lengths of pipe in the system, rather than loss in volume through fixtures. Per one hundred feet, there is a fractional decrease in pressure, ΔH, via inches in water column. Figure 1.24 displays a Table for indicating pipe diameter as a function of pressure loss.

PRESSURE DROP PER 100 FEET IN INCHES W.C.	PIPE SIZES (inch)					
	$^1/_2$	$^3/_4$	1	$1^1/_4$	$1^1/_2$	2
0.2	31	64	121	248	372	716
0.3	38	79	148	304	455	877
0.5	50	104	195	400	600	1160
1.0	71	147	276	566	848	1640

Figure 1.24: Sample gas distribution schematic. (*2010 Oregon Mechanical Specialty Code. Table C-A.3.4. Thousands of Btu/h (MBTU) of Natural Gas per 100 Feet of Pipe at Various Pressure Drops and Pipe Diameters*)

Finally, one can use the **direct application of Equations** (1.3), (1.4) for low- or high-pressure gas. After all, these are responsible for all other sizing methods and the associated charts and tables.

In many situations, natural gas may be treated as *ideal gas*, which could be useful in the following scenario: If a closed piping system is installed on a warm afternoon but tested on a chilled morning, the pressure will have decreased according to the ideal gas law:

$$PV = NKT$$

With V, N, K constant, it follows that the pressure-temperature relationship can be written

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}\,.$$

3 Venting

3.1 The 5-6-8-10 Rule

When attaching a plumbing fixture to a vent pipe, the distance from that fixture's trap weir to the vent connection cannot exceed a certain length, depending on the diameter of the pipe being used.

CMR 10.16 provides information on the matter as shown in Figure 1.25. Despite the typo in Table 1, we glean that, for instance, a pipe diameter of two inches cannot exceed six feet in length as sketched. Reading down the right column, this has been named the 5-6-8-10 rule.

Table 1
Distance of Fixture Trap from Vent

Diameter of Pipe	Maximum Developed Length of the Pipe
1½	5"
2	6'
3	8'
4	10'
Slope not to exceed ¼" per foot	

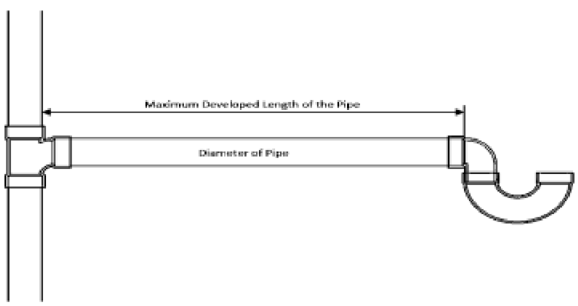


Figure 1.25: CMR 10.16 Table 1 with provided sketch.

While the sketch provided under Table 1 means well, the drain pipe shown is perfectly horizontal and does not illustrate the point of the rule. To remedy this, imagine tilting the pipe toward the vent by an angle θ with respect to the horizontal as shown in Figure 1.26.

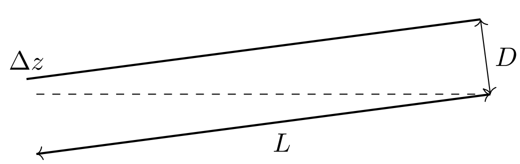


Figure 1.26: Sloped pipe.

By convention, the angle θ for pipes of diameter ≤ 3 in corresponds to a minimum uniform pitch of $1/4$ in per foot. For pipes greater than three inches, the minimum uniform pitch is $1/8$ in per foot. Figure 1.26 exaggerates this angle for clarity.

Paying attention to the dashed horizontal line, one sees that adjusting the angle θ of the pipe will change the height Δz. This is the vertical distance from the trap weir to the top of the vent connection.

For simultaneous drainage and venting through the pipe, it makes sense that Δz should never be zero or negative. Moreover, Δz should be far enough from zero so that venting is maintained.

From trigonometry, observe in Figure 1.26 that

$$L \sin(\theta) = D - \Delta z \, ,$$

where L is the pipe length and D is the diameter. As mentioned, θ takes one of two branches:

$$\theta = \begin{cases} 1/4 \text{ in} \cdot \text{ft}^{-1} & \arctan(1/48) \approx 0.0208 \\ 1/8 \text{ in} \cdot \text{ft}^{-1} & \arctan(1/96) \approx 0.0104 \end{cases}$$

With this, we set up four calculations for Δz (converting feet so Δz comes out in inches):

$$\begin{aligned} \Delta z_{1.5} &= (1.5 - 60 \sin(0.0208)) \text{ in} \\ \Delta z_2 &= (2 - 72 \sin(0.0208)) \text{ in} \\ \Delta z_3 &= (3 - 96 \sin(0.0208)) \text{ in} \\ \Delta z_4 &= (4 - 120 \sin(0.0104)) \text{ in} \end{aligned}$$

Crunching the numbers, one finds:

$$\begin{aligned} \Delta z_{1.5} &= 0.2521 \text{ in} \\ \Delta z_2 &= 0.5025 \text{ in} \\ \Delta z_3 &= 1.003 \text{ in} \\ \Delta z_4 &= 2.752 \text{ in} \end{aligned}$$

Looking for some order in the above, we see the first three Δz roughly doubling with each increase in pipe size. The four-inch case breaks this pattern, as $2\Delta z_3$ is not 2.752 inches.

As a ratio to pipe diameter, we also learn:

$$\begin{aligned} \Delta z_{1.5}/1.5 \text{ in} &\approx 17\% \\ \Delta z_2/2 \text{ in} &\approx 25\% \\ \Delta z_3/3 \text{ in} &\approx 33\% \\ \Delta z_4/4 \text{ in} &\approx 69\% \end{aligned}$$

These figures are a bit deceiving in the sense that each percentage on the right represents a ratio of lengths, not a ratio of areas.

More interesting than the ratio $\Delta z/D$ would instead be the cross-sectional area represented by Δz, i.e. the air gap at the end of the pipe divided by the total cross-sectional area $\pi D^2/4$. Leaving the details aside, it's possible to show the ratio R of areas to be:

$$R\,(17\%) \approx 11\%$$
$$R\,(25\%) \approx 20\%$$
$$R\,(33\%) \approx 29\%$$
$$R\,(69\%) \approx 74\%$$

We can make the most informed conclusion from the right side of the above results. At the vent connection, we conclude that a 1.25 in pipe needs about 10% of its cross-sectional area open for venting. Going up in diameter, the two-inch pipe needs 20% open area, the three-inch pipe needs 30% open area.

For the four-inch pipe, we see nearly three quarters of the pipe is reserved for venting, presumably because such pipes tend to carry large volumes of solids.